Multi-stage taxation by subnational governments: Tax incidence and Leviathan taxation*

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Abstract

This paper analyzes multi-stage taxation by provinces in a federal country, using a novel two-good, two-province, successive-oligopoly heterogeneous-product Bertrand competition model, where each producer is located in a province and sells its product through exclusive retailers located in both provinces. The producer-retailer setup allows provincial governments to raise taxes on upstream and downstream transactions. First, we analyze tax incidence results and emphasize the importance of the degree of downstream competition on the tax shifting. Second, we solve a non-cooperative revenue maximization problem and study the properties of the equilibrium taxes. We characterize the solution: either all tax rates are positive or one province drops one tax rate. This way, the full-tax solution dominates upstream and downstream taxation. Also, the non-cooperative solution implies over-taxation when compared with the cooperative solution.

Este paper analiza impuestos en múltiples etapas recaudados por provincias en un país federal, utilizando un modelo novedoso de competencia en dos segmentos productor-minorista a la Bertrand con productos diferenciados, donde cada productor está ubicado en una provincia y vende en todo el país a través de sus distribuidores. Este modelo genera la posibilidad de que las provincias recauden en las dos etapas de transacciones. En primer lugar, se obtienen resultados de incidencia, enfatizando la importancia del grado de diferenciación sobre la traslación. En segundo lugar, se resuelve el problema de maximización de recaudación en un contexto no cooperativo y se estudian las propiedades de las alícuotas de equilibrio: las provincias utilizan todas las alícuotas o una de ellas descarta sólo una alícuota. La solución no cooperativa domina (en términos de recaudación) a los casos de impuestos al productor o a las ventas minoristas. Por otro lado, esta solución se caracteriza por imposición excesiva (respecto de un caso cooperativo).

Keywords: local indirect taxation, multistage taxes, tax icidence, tax competition.

JEL Codes: H71, H21, H22.

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1 Introduction

General taxes on goods and services (TGS) are an important source of fiscal revenue. They are top–of–the–list within indirect taxes, and are typically classified into value-added –or consumption– taxes, sales taxes, multi-stage cumulative taxes, excises, taxes on trade, etc. (following OECD's classification). Other important sources of revenue are taxes on income (direct taxation on corporate or personal income or earnings) and property (direct taxation on wealth).

When the power to raise revenues is vested on a centralized level, fiscal authorities prefer VAT or sales taxes over multi-stage taxes. There is a reason for that: multi-stage taxation creates inefficiency along the value-chain (the so-called "cascade" effect).[1] Governments naturally internalize the double-margin effect from multi-stage taxation and prefer consumption or sales taxes over turnover taxes. In fact, countries that joined the European Community used to raise some form of turnover taxes, and replaced them for VAT during the mid-80s (Tait, 1988).

But multi-stage taxes arise as a preferred instrument at provincial levels, although they are not widespread. There is also a reason for that: provinces may have incentive to export or import tax bases. For example, in Argentina provincial governments collect turnover taxes (*impuesto sobre los ingresos brutos*), which represent –in average– more than 40 percent (60 percent in some provinces) of provincial resources.[2]

This paper analyzes multi-stage taxation by provinces in a federal country. We set up

[1] The cascade effect has long been studied by the public finance literature. See Friedlaender (1967).

[2] We refer the reader to many papers of this tax in Argentina. See, for example, Núñez Miñana (1994), Libonatti (1998), Porto, Garriga and Rosales (2014) and the references therein.

a two-good, two-province, successive-oligopoly heterogeneous-product Bertrand competition model, where each producer is located in a province and sells its product through exclusive retailers located in both provinces. The producer-retailer setup allows provincial governments to raise taxes on both upstream and downstream links of the value chain. With this model, we study tax incidence and solve a taxation problem with provinces that seek to maximize tax revenues (Leviathan) non-cooperatively. We analyze the properties of the equilibrium taxes and compare them to specific cases (upstream and downstream non-cooperative taxation, and cooperative taxation). Although the setup and results apply to provinces, they extend straightforwardly to countries coordinated within a Union (such as the European Union or Mercosur), in which governments may export / import bases to taxes collected at federal level.

The following results are obtained in this paper. First, we study the tax incidence properties of the model, analyzing changes in producer and retailer prices due to changes in upstream or downstream tax rates, and showing that these incidence effects depend crucially on the degree of product differentiation and therefore on the level of (downstream and upstream) market power. Second, assuming provinces that maximize tax revenues (Leviathan) non-cooperatively, we analyze tax rates in equilibrium under different scenarios. In the symmetric case (equal market sizes, retail costs and producer costs for both products) with no competition at the downstream level, we find a double-tax-rate mark-up result –which mimics the standard producer-retailer relationship obtained in the industrial organization literature– with upstream rates doubling downstream rates in equilibrium. As downstream competition gets stronger, provincial governments gradually switch from upstream to downstream taxation, eventually abandoning upstream taxation completely when products are homogeneous. Different asymmetric configurations of relevant parameters (market size, regional retail costs, producer costs or retail costs by product) translate in different asymmetries in upstream and downstream taxes in equilibrium. A general result (proved in separate propositions in the paper) emerges: if parameters are such that equilibrium exists and products exhibit a certain degree of heterogeneity, provinces will choose positive tax rates at both levels (upstream and downstream) or at most one of the four tax rates is zero. As in the symmetric case, only downstream tax rates are positive under product homogeneity, although at different rates between regions. Third, we obtain a dominance result in terms of provincial tax revenue, provided goods are heterogeneous: successive taxation raises higher revenues over cases in which governments are restricted to use either upstream (production) or downstream (consumption) taxation. Only when products are homogeneous the full taxation and downstream taxation are revenue-equivalent. This result provides a strong ground for using successive taxation (over production- or consumption-based taxation) when provincial governments need to raise resources.[3] Finally, we compare full-taxation in a non cooperative setting with a coordinated scenario where provincial governments choose tax rates to maximize joint revenues and obtain an over-taxation result: if there is a minimum degree of product differentiation, non-cooperative taxation implies over-taxation, and hence less revenues, compared with cooperative taxation. Only when products are homogeneous the non-cooperative and cooperative solutions are revenue-equivalent.

The paper links to different branches of the literature. First, tax incidence under different market structures has been analyzed extensively in the literature, although not in a context of (successive) taxation by subnational governments (Bishop, 1968, Anderson, de Palma and Kreider, 2001, and Fullerton and Metcalf, 2002).

Second, Peitz and Reisinger (2014) study taxation in successive-markets, but they focus on the per-unit ad-valorem tax mix, whereas this paper analyzes the upstream-downstream tax mix. Moreover, we define a short-run price competition model with product differentiation and a pre-set location of producer in different provinces, while Peitz and Reisinger (2014) study the successive taxation problem in an homogeneous quantity-competition (Cournot) oligopoly with entry but without geographic concerns.

[3] A companion paper studies successive taxation following welfare objectives in a simpler setting. Under certain configuration of parameters on preferences, market sizes and costs the solution implies successive taxation.

Third, there is a link with the literature on origin or destination principles for taxation (Lockwood 1993, 2001, Keen and Lahiri, 1993, 1998). Keen and Lahiri (1993) focus on harmonization of destination-based taxes. Keen and Lahiri (1998) focus on welfare consequences of switching between destination and origin principles when (i) both taxes are fixed, (ii) both taxes are optimized. Keen (1989) and Lockwood (1997) analyze tax harmonization and Pareto-improving reforms when goods are heterogeneous. Trandel (1994) focuses on interstate commodity tax. Lockwood (2001) analyzes tax competition under origin and destination principles, based on consumer price spillover, producer price/terms of trade spillover and rent spillover, but leaves aside multi-stage taxation.

A fourth link exists with the literature on the distortions generated by taxes imposed on several levels of the value chain, like the turnover tax (the “cascade effect” mentioned above). The contribution of this paper is that “the cascade effect” trades off with the possibility of importing or exporting tax bases, which is relevant at provincial levels.[4]

Finally, we build a differentiated-product successive-oligopoly model, where we assume product differentiation using the demand proposed by Deneckeree and Davidson (1985). The choice of a demand function that captures product differentiation in a simple way (together with linear costs) allows us to get closed-form solutions for market competition and tax rates.[5]

The rest of the paper is organized as follows. Section 2 introduces a novel competition model and characterizes the equilibrium prices and quantities at retail and producer levels. Section 3 studies tax incidence results, with emphasis on the relationship with the degree of product differentiation and market power. Section 4 solves the full-tax non-cooperative Leviathan problem, characterizes the solution, and compares it with a scenario where provincial governments are restricted from using only one instrument. Section 5 explores

[4] Das-Gupta (2005) constructs an example where the turnover tax can dominate the VAT under a successive monopoly in terms of welfare, revenue and output.

[5] See Inderst and Valletti (2011) and Reisinger and Schnitzer (2012) regarding modeling upstream-downstream markets with differentiated products.

the cooperative solution and compares it with the non-cooperative case. Finally, Section 6 concludes.

2 The Model

2.1 Setup

Assume two producers of goods A and B. Producer A is located in country / province 1, producer B is located in region / province 2, and they do not re-locate across countries / provinces.[6] In order to reach consumers, producers sell their products to retailers in an *upstream* market. Retailers sell products to consumers in a *downstream* market. The structure is as follows.

Producer A (located in region 1) sells products to retailer $1A$ in region 1 (x_{1A}) and retailer $2A$ in region 2 (x_{2A}) at price p_A. Total quantity of product A is $x_A = x_{1A} + x_{2A}$. Likewise, producer B (located in region 2) sells products to retailer $1B$ in region 1 and retailer $2B$ in region 2 at price p_B. Total quantity of product B is $x_B = x_{1B} + x_{2B}$. Therefore, we rule out third-degree (regional) price discrimination. Producers' marginal cost is constant and equal to c_A and c_B, respectively.

Retailers in market $i = 1, 2$ sell to consumers with demand

$$x_{ij} = a_i - p_{ij} - \gamma\left(p_{ij} - p_{ik}\right) \tag{1}$$

where subscript i is region and j, k correspond to products (either A, B or B, A). We use a simple demand (1) (as that used by in Deneckeree and Davidson, 1985, to study merger among firms competing with differentiated products) to characterize the equilibrium based

[6] This paper does not analyze location decisions. It may be the case that firms locate in a country or province following costs or tax advantages ("fiscal benefits"). Once investment decisions are made, there is a time span –the relevant time throughout this paper– in which production plants stay in the country / province.

on market size (a_i) and the degree of market-power linked to product differentiation (γ). Retailers' costs are

$$CT_{ij}(x_{ij}) = (p_j + c_{ij})\, x_{ij}$$

There are regional / provincial governments that collect sales taxes. They can set rates on upstream sales (τ_i) and / or downstream sales (t_i) within their jurisdiction. Tax revenues in provinces 1 and 2 are

$$\begin{aligned} R_1 &= \tau_1 x_A + t_1\,(x_{1A} + x_{1B}) = (\tau_1 + t_1)x_{1A} + t_1 x_{1B} + \tau_1 x_{2A} \\ R_2 &= \tau_2 x_B + t_2\,(x_{2A} + x_{2B}) = (\tau_2 + t_2)x_{2B} + t_2 x_{2A} + \tau_2 x_{1B} \end{aligned}$$

Revenues are collected on upstream and downstream sales of units produced and sold within the same province, upstream sales of units produced in the province but sold in the other province, and downstream sales of units produced in the other province and sold in the government's province. With this setup, Section 2.2 characterizes the market equilibrium upon which the local governments collect taxes.

2.2 Producers and retailers: equilibrium and profits

2.2.1 Retailers equilibrium

Let the subindex ij (ik) stand for product j in province i (product k –substitute to product j– in province i). Also, let $\bar{c}_{ij} = c_{ij} + t_i + p_j$. Retailers' profit is

$$\pi_{ij} = (p_{ij} - \bar{c}_{ij})\,[a_i - p_{ij} - \gamma(p_{ij} - p_{ik})]$$

Profit maximization and equilibrium at the downstream level lead to the following prices and quantities

$$\begin{aligned} p_{ij} &= \frac{a_i}{(2+\gamma)} + \frac{(1+\gamma)\,[2(1+\gamma)\bar{c}_{ij} + \gamma\bar{c}_{ik}]}{(2+\gamma)(2+3\gamma)} \\ x_{ij} &= \frac{(1+\gamma)(a_i - c_{ij} - t_i - p_j)}{(2+\gamma)} + \frac{\gamma(1+\gamma)^2\,(c_{ik} + p_k - c_{ij} - p_j)}{(2+\gamma)(2+3\gamma)} \end{aligned} \qquad (2)$$

2.2.2 Producers equilibrium

Producer A's profit is $\pi_A = (p_A - \tau_1 - c_A)[x_{1A} + x_{2A}]$, i.e.,

$$\pi_A = (p_A - \tau_1 - c_A)\left[\begin{array}{c} \frac{(1+\gamma)(a_1-c_{1A}-t_1-p_A)}{(2+\gamma)} + \frac{\gamma(1+\gamma)^2(c_{1B}+p_B-c_{1A}-p_A)}{(2+\gamma)(2+3\gamma)} + \\ \frac{(1+\gamma)(a_2-c_{2A}-t_2-p_A)}{(2+\gamma)} + \frac{\gamma(1+\gamma)^2(c_{2B}+p_B-c_{2A}-p_A)}{(2+\gamma)(2+3\gamma)} \end{array}\right]$$

Producer B's profit is $\pi_B = (p_B - \tau_2 - c_B)[x_{1B} + x_{2B}]$, i.e.,

$$\pi_B = (p_B - \tau_2 - c_B)\left[\begin{array}{c} \frac{(1+\gamma)(a_1-c_{1B}-t_1-p_B)}{(2+\gamma)} + \frac{\gamma(1+\gamma)^2(c_{1A}+p_A-c_{1B}-p_B)}{(2+\gamma)(2+3\gamma)} + \\ \frac{(1+\gamma)(a_2-c_{2B}-t_2-p_B)}{(2+\gamma)} + \frac{\gamma(1+\gamma)^2(c_{2A}+p_A-c_{2B}-p_B)}{(2+\gamma)(2+3\gamma)} \end{array}\right]$$

Profit maximization and equilibrium at the upstream level leads to the following solution:

$$p_A = \Phi_A - \delta(t_1+t_2) + \theta\tau_2 + \omega\tau_1 \qquad p_B = \Phi_B - \delta(t_1+t_2) + \theta\tau_1 + \omega\tau_2 \tag{3}$$

$$x_{1A} = \Gamma_{1A} - \alpha t_1 + \beta t_2 - \mu\tau_1 + \kappa\tau_2 \qquad x_{1B} = \Gamma_{1B} - \alpha t_1 + \beta t_2 + \kappa\tau_1 - \mu\tau_2 \tag{4}$$

$$x_{2A} = \Gamma_{2A} + \beta t_1 - \alpha t_2 - \mu\tau_1 + \kappa\tau_2 \qquad x_{2B} = \Gamma_{2B} + \beta t_1 - \alpha t_2 + \kappa\tau_1 - \mu\tau_2 \tag{5}$$

$$p_{1A} = \Psi_{1A} + \alpha t_1 - \beta t_2 + \rho\tau_1 + \sigma\tau_2 \qquad p_{1B} = \Psi_{1B} + \alpha t_1 - \beta t_2 + \rho\tau_2 + \sigma\tau_1 \tag{6}$$

$$p_{2A} = \Psi_{2A} + \alpha t_2 - \beta t_1 + \rho\tau_1 + \sigma\tau_2 \qquad p_{2B} = \Psi_{2B} + \alpha t_2 - \beta t_1 + \rho\tau_2 + \sigma\tau_1 \tag{7}$$

Constants Φ_A, Φ_B are the no-tax producer prices (from equations (22)-(23) in Appendix A); $\Psi_{1A}, \Psi_{1B}, \Psi_{2A}, \Psi_{2B}$ are the no-tax retail prices (from equations (24)-(27) in Appendix A); $\Gamma_{1A}, \Gamma_{1B}, \Gamma_{2A}, \Gamma_{2B}$ are the no-tax equilibrium quantities (from equations (28)–(31) in Appendix A); and $(\alpha, \beta, \mu, \kappa, \rho, \sigma, \delta, \theta, \omega)$ are parameters –which depend on γ– defined in equations (32)-(37). We discuss the assumptions on parameters for an interior solution in Appendix B.

3 Incidence analysis

3.1 General results

Tax incidence in this model is about determining the effect of changes in upstream or downstream tax rates on both producers and retailers prices.[7] These effects, and the corresponding ones for quantities, can be readily calculated from equations (3)-(7). As it can be anticipated, results depend crucially on the value of γ, which reflects the degree of product differentiation and hence the level of (downstream and upstream) market power.

Incidence effects of upstream taxes. An increase in an upstream tax rate affects producer and retailer prices as follows:

$$\frac{dp_j}{d\tau_i} = \omega \qquad ; \qquad \frac{dp_k}{d\tau_i} = \theta \qquad ; \qquad \frac{dp_{ij}}{d\tau_i} = \rho \qquad ; \qquad \frac{dp_{ij}}{d\tau_h} = \sigma$$

where j refers to producer in region i and k refers to producer in the other region (labeled h below).

Incidence effects of downstream taxes. Similarly, an increase in a downstream tax rate affects producer and retailer prices as follows:

$$\frac{dp_j}{dt_i} = \frac{dp_j}{dt_h} = -\delta \qquad ; \qquad \frac{dp_{ij}}{dt_i} = \alpha \qquad ; \qquad \frac{dp_{ij}}{dt_h} = -\beta$$

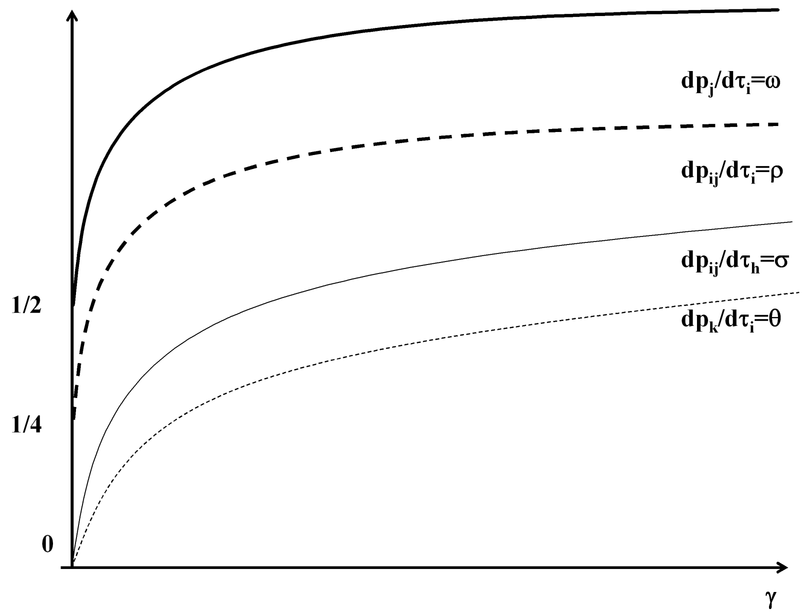
Figures 1 and 2 show these effects for different values of γ.

3.2 Specific results: cases of $\gamma = \{0, 1, \infty\}$

Table 1 shows the case with $\gamma = 0$ (left panel), an intermediate case with positive γ ($\gamma = 1$, represented in the middle panel) and the extreme case with $\gamma \to \infty$ (right panel).

[7] There is a vast literature on tax incidence under different market structures, with emphasis on the comparison between ad valorem versus per unit taxes in terms of efficiency and the degree of forward shifting. We refer the reader to Fullerton and Metcalf (2002) for a general discussion.

Figure 1: **Tax incidence: producer tax rates**

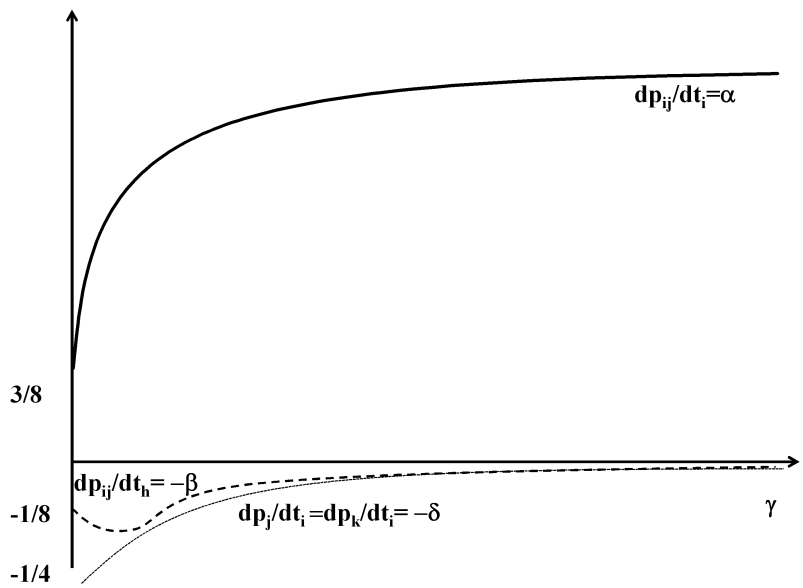


<u>Case: $\gamma = 0$.</u> In this case there is no downstream competition in each region, so the incidence results are the expected from a situation with two separate monopolies in both regions, each one characterized by the standard producer-retailer relationship and the double marginalization described in the industrial organization literature (see Tirole, 1988, p.174; see also Bishop, 1968, regarding the shifting effect of a per unit tax in a monopoly). Results are summarized in the left panel of Table 1.

Consider first increasing upstream taxes in one region. An increase in τ_i (say, τ_1) passes-through producer price in region i (product A) by half, and through retail prices (i.e., p_{1A} and p_{2A}) by one fourth, the increase in τ_i. On the other hand, the increase in τ_i does not affect the producer price p_B (produced in province h) nor their corresponding retail prices (in the example, p_{1B} and p_{2B}).

Next, consider increasing downstream tax rates. An increase in t_1 (in region 1) induces backward-shifting for both producer prices sold in province 1 (p_A and p_B, by -1/4 the

Figure 2: **Tax incidence: retail tax rates**



increase in t_1), and forward-shifting for prices of final products sold in the same region (p_{1A} and p_{1B}, by 3/8 the increase in t_1). Moreover, the reduction in producer prices is partially passed-through retail prices in the other region (p_{2A} and p_{2B}, by 1/8 the increase in t_1).

Case: $\gamma = 1$. Next, consider the case that products are subject to some competition ($\gamma = 1$).

An increase in τ_1 passes through both retail prices (p_{1A} and p_{2A}) and producer price p_A, just as in the previous case with $\gamma = 0$. But in this case, product $1B$ (resp. $2B$) compete with $1A$ (resp. $2A$) given the model of price competition set out in Section 2. Since prices are strategic complements, it will follow an increase in p_{1B} and p_{2B} (and corresponding reactions in p_{1A} and p_{2A}). Moreover, p_B is also strategic complement to p_A and therefore both producer prices will settle in a higer level.[8]

[8] Comparing the left and middle panels of Table 1, the direct effect of τ_1 on p_A is 0.5 and the indirect

Table 1: **Tax incidence. Specific cases**

	$\gamma=0$				$\gamma=1$				$\gamma\to\infty$			
	t_1	t_2	τ_1	τ_2	t_1	t_2	τ_1	τ_2	t_1	t_2	τ_1	τ_2
p_A	-1/4	-1/4	1/2	0	-0.21	-0.21	0.51	0.07	0	0	2/3	1/3
p_B	-1/4	-1/4	0	1/2	-0.21	-0.21	0.07	0.51	0	0	1/3	2/3
p_{1A}	3/8	-1/8	1/4	0	0.53	-0.14	0.28	0.11	1	0	5/9	4/9
p_{1B}	3/8	-1/8	0	1/4	0.53	-0.14	0.11	0.28	1	0	4/9	5/9
p_{2A}	-1/8	3/8	1/4	0	-0.14	0.53	0.28	0.11	0	1	5/9	4/9
p_{2B}	-1/8	3/8	0	1/4	-0.14	0.53	0.11	0.28	0	1	4/9	5/9
x_{1A}	-3/8	1/8	-1/4	0	-0.53	0.14	-0.46	0.07	-1	0	-∞	∞
x_{1B}	-3/8	1/8	0	-1/4	-0.53	0.14	0.07	-0.46	-1	0	∞	-∞
x_{2A}	1/8	-3/8	-1/4	0	0.14	-0.53	-0.46	0.07	0	-1	-∞	∞
x_{2B}	1/8	-3/8	0	-1/4	0.14	-0.53	0.07	-0.46	0	-1	∞	-∞

Next, consider increasing downstream tax rates. An increase in t_1 (in region 1) induces backward-shifting for both producer prices sold in province 1, forward-shifting for prices of final products sold in the same region, and pass through to retail prices in the other region. But a second-order effect enters into effect as final products are substitutes and retail prices strategically interact: the increase in p_{1A} and p_{1B} exceeds 3/8 to internalize cross-effects (this is the main effect), pushing producer prices –and hence retail prices in the other region– upwards. However, the second order effect is not strong enough to overcome the main effect (described with $\gamma = 0$). As γ increases, the second-order effect pushes upwards retail prices in the provinces that increases retail tax rate (up to full pass through), and fully compensate the first-order effect of backwards shifting for producer prices and retail prices in the other province.

effect is 0.01, while the indirect effect on p_B is 0.07. The direct effect of τ_1 on p_{1A} and p_{2A} is 0.25 and the indirect effect is 0.03, while the indirect effect on p_{1B} and p_{2B} is 0.11.

Case: $\gamma \to \infty$. Finally, this case reflects a market in which products are subject to intense competition (in the extreme, final products are perfect substitutes). Assume that producers and retailers are symmetric on the cost side.

An increase in τ_1 causes that retail cost of product A increases in both regiones, and they cannot compete in an homogeneous-product market. As a consequence, only product B will be sold in both regions. On the other hand, an increase in downstream tax rate (t_1) produces full shifting results in the corresponding region (p_{1A} and p_{1B}) and null effects on producer prices (see Bishop, 1968, for shifting of per unit taxes under perfect competition, and Fullerton and Metcalf, 2002, for shifting of per unit taxes under Bertrand competition). Consequently, retail prices in the other region remain unaltered.

4 Provinces - Revenue maximization

In this section we assume that provinces maximize tax revenue. Pure tax collection is the natural starting point to analyze why governments choose taxing several links in the value chain. In another paper we deal with the same problem when provinces have different objective, specifically, a local welfare function.

With the information from Section 2.2, provincial tax revenues are

$$
\begin{aligned}
R_1 &= (\tau_1 + t_1)x_{1A} + t_1 x_{1B} + \tau_1 x_{2A} \\
&= (\tau_1 + t_1)\left(\Gamma_{1A} - \alpha t_1 + \beta t_2 - \mu\tau_1 + \kappa\tau_2\right) + t_1\left(\Gamma_{1B} - \alpha t_1 + \beta t_2 + \kappa\tau_1 - \mu\tau_2\right) \\
&\quad + \tau_1\left(\Gamma_{2A} + \beta t_1 - \alpha t_2 - \mu\tau_1 + \kappa\tau_2\right) \\
R_2 &= (\tau_2 + t_2)x_{2B} + t_2 x_{2A} + \tau_2 x_{1B} \\
&= (\tau_2 + t_2)\left(\Gamma_{2B} + \beta t_1 - \alpha t_2 + \kappa\tau_1 - \mu\tau_2\right) + t_2\left(\Gamma_{2A} + \beta t_1 - \alpha t_2 - \mu\tau_1 + \kappa\tau_2\right) + \\
&\quad \tau_2\left(\Gamma_{1B} - \alpha t_1 + \beta t_2 + \kappa\tau_1 - \mu\tau_2\right)
\end{aligned}
$$

4.1 Full taxation

When governments collect upstream and downstream taxes, the first-order conditions for tax rates are

$$t_1 \quad : \quad \Gamma_{1A} + \Gamma_{1B} - 4\alpha t_1 + 2\beta t_2 - 2\psi\tau_1 - \psi\tau_2 = 0 \tag{8}$$

$$\tau_1 \quad : \quad \Gamma_{1A} + \Gamma_{2A} - 4\mu\tau_1 + 2\kappa\tau_2 - 2\psi t_1 - \psi t_2 = 0 \tag{9}$$

$$t_2 \quad : \quad \Gamma_{2A} + \Gamma_{2B} - 4\alpha t_2 + 2\beta t_1 - 2\psi\tau_2 - \psi\tau_1 = 0 \tag{10}$$

$$\tau_2 \quad : \quad \Gamma_{2B} + \Gamma_{1B} - 4\mu\tau_2 + 2\kappa\tau_1 - 2\psi t_2 - \psi t_1 = 0 \tag{11}$$

Result 1 *Reaction functions for government i's downstream (upstream) tax rates are strategic complements with respect government j's downstream (upstream) tax rates and strategic substitutes with respect to government j's upstream (downstream) tax rates.*

This result states that a provincial government reduces the tax burden on retail sales when the other government increases upstream rates or increases retail rates. On the other hand, a government reduces upstream rates when the other government increases downstream rates or decreases upstream rates.

Lemma 1 *If $t_2 = \tau_2 = 0$, government 1's tax rates are $t_1(0,0) > 0$ and $\tau_1(0,0) > 0$, and viceversa.*

Proof: Working with government 1's reaction functions we get

$$t_1 = \frac{(2\mu - \psi)(\Gamma_{1A} + \Gamma_{1B})}{2(4\alpha\mu - \psi^2)} > 0 \quad \text{and} \quad \tau_1 = \frac{(2\alpha - \psi)(\Gamma_{1A} + \Gamma_{2A})}{2(4\alpha\mu - \psi^2)} > 0$$

Q.E.D.

If a government does not collect taxes, the best response by the other government is to set positive upstream and downstream taxes. This sheds light on the main results of this section, i.e., of taxing at both levels of economic transactions whenever possible.

The solution to conditions (8)-(11) is reported in Appendix B. Equations (38)-(41) correspond to the interior solution; equations (46)-(48) correspond to the case when $t_1^* = 0$; equations (49)-(51) when $t_2^* = 0$; equations (52)-(54) when $\tau_1^* = 0$; and equations (55)-(57) when $\tau_2^* = 0$.

As it can be seen from those equations, the solution depends on the combination of nine parameters $(a_1, a_2, c_{1A}, c_{1B}, c_{2A}, c_{2B}, c_A, c_B, \gamma)$.Rather than following this path, we develop a sequence of propositions on tax rates in equilibrium under different scenarios. First, we develop the case of symmetric demands and costs.

Proposition 1 *Symmetric solution. Let $\Lambda = a - c_R - c_P$ and $\Gamma_{1A} = \Gamma_{1B} = \Gamma_{2A} = \Gamma_{2B} = \Gamma = \psi\Lambda$. Equilibrium rates are $t_1^S = t_2^S = t^S$ satisfying (12) and $\tau_1^S = \tau_2^S = \tau^S$ satisfying (13).*

$$t^S = \frac{\Gamma(2\mu - \psi)}{4(2\alpha - \beta)(2\mu - \kappa) - 9\psi^2} \tag{12}$$

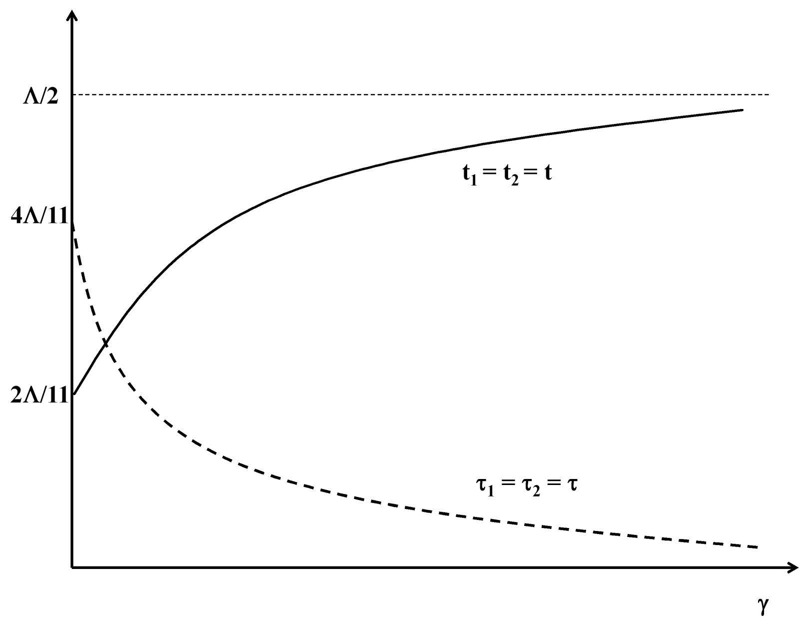
$$\tau^S = \frac{\Gamma(2\alpha - \psi)}{4(2\alpha - \beta)(2\mu - \kappa) - 9\psi^2} \tag{13}$$

Proof: See Appendix B.1.

When $\gamma = 0$ we see that all rates are positive, with upstream rates doubling downstream rates; specifically, $t^S(\gamma = 0) = 2\Lambda/11$ and $\tau^S(\gamma = 0) = 2t^S(\gamma = 0)$. As γ increases, upstream rates decrease while the burden switches to retail taxation. Finally, when products are homogeneous governments give up upstream taxation and concentrate on retail taxation, i.e., $t^S(\gamma \to \infty) = \Lambda/2$ and $\tau^S(\gamma \to \infty) = 0$.

Recall that in the case of $\gamma = 0$, absent taxation, upstream mark-ups double downstream mark-ups (this is the standard double-mark up result in a producer-retailer relationship without competition; see Tirole, 1998, chapter 4). It is straightforward to understand that a provincial government –whose objective is to raise tax revenue– will

Figure 3: **Equilibrium upstream and downstream tax rates under symmetry**



follow the private solution and set tax rates in a similar way as firms set the price structure. Therefore, $\tau = 2t$ and there is a double-tax-rate-mark up result.

As the degree of product differentiation decreases (higher γ) downstream competition gets stronger. Governments switch from upstream to downstream taxation, i.e., τ decreases and t increases in γ, as shown in Figure 3. In the limiting case of homogeneous products at the downstream level, there is no room to collect taxes in the upstream. If a provincial government (say 1) increases the upstream rate (τ_A) it imposes a disadvantage on product A in both provinces (by artificially increasing retail cost) to the point of driving it out of both markets, and is left to collect retail tax on the good produced in the other province. On the other hand, when the government raises downstream taxes, it does so uniformly on all retailers selling products in the province. Since downstream taxation does not create asymmetries on retail competition, there is room to collect downstream taxes.

Moreover, as the degree of differentiation is reduced tax revenues increase because tax bases increase for both provinces. In other words, the higher the γ the lower the markup that retailers and producers set on their products, and the higher the quantities for given market sizes ($\Lambda = a - c_R - c_P$). It follows that provincial collect higher tax revenues.

In the remainder of the section we focus on specific asymmetries. The next four propositions, each one accompanied by a Figure, share the following pattern. First, we consider cases such that, in the absence of taxes, retailers and producers are active. In each Figure, a *dotted* line will identify the set of parameters that satisfy this condition (leaving aside a set of ruled-out parameters; see Appendix A). Second, the selected asymmetry may induce governments to switch from full taxation (i.e., collecting upstream and downstream taxes) to partial taxation (i.e., foregoing either upstream or downstream taxes). Therefore we identify a threshold in parameters that divide the cases of full or partial taxation (a dashed line). Third, under either asymmetry (parameters or equilibrium tax rates) producer or retailer mark ups may become negative. In order to keep the paper tractable, we assume parameters such that all markets –wholesale and retailer– remain open in the tax equilibrium (a possibly stringent constraint than the first one detailed here).[9] Therefore we identify the threshold in parameters for non-negative mark ups (a *solid* line). Using both thresholds, we develop the following results.

In the first case, we assume asymmetries in market size.

Proposition 2 *Asymmetries in market size. Assume symmetric cost structures ($c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$ and $c_A = c_B = c_P$) and let $a_2 \geq a_1$. There are two functions $\underline{a}_2/a_1(\gamma)$ and $\bar{a}_2/a_1(\gamma)$, continuous and increasing, that intersect at $\hat{\gamma}_{a2}$. If $\gamma < \hat{\gamma}_{a2}$, equilibrium taxes are such that $\{t_2 > t_1 > 0; \tau_1 > \tau_2 > 0\}$ if $a_2/a_1 \leq \underline{a}_2/a_1(\gamma)$ and $\{t_1 = 0; t_2 > 0; \tau_1 > \tau_2 > 0\}$ if $\underline{a}_2/a_1(\gamma) < a_2/a_1 \leq \bar{a}_2/a_1(\gamma)$. If $\gamma \geq \hat{\gamma}_{a2}$, equilibrium taxes*

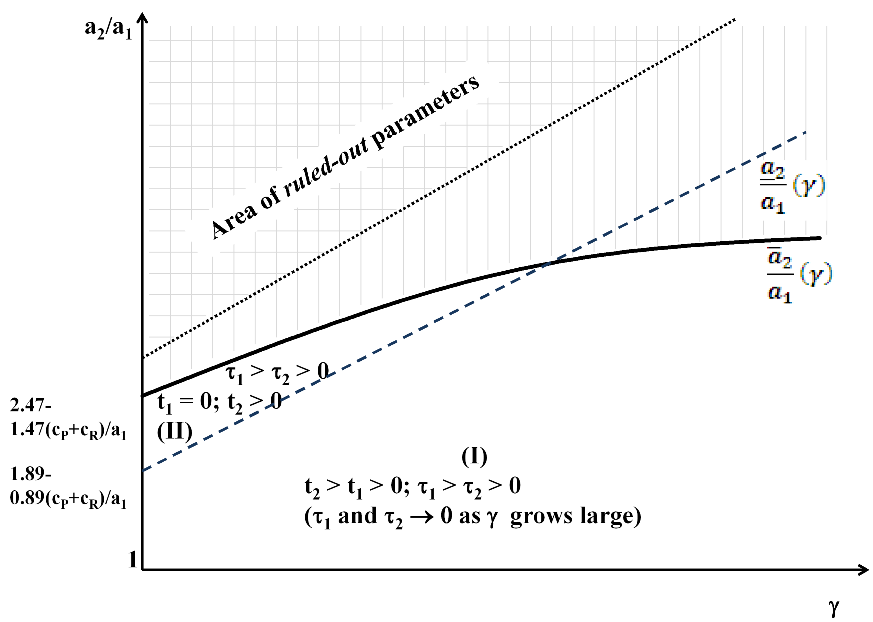
[9] When asymmetries are sufficiently large, it is possible that a retailer or producer closes down the market, leaving the other retailer or producer as a (retailer or wholesale) monopolist.

are such that $\{t_2 > t_1 > 0; \tau_1 > \tau_2 > 0\}$ *if* $a_2/a_1 \leq \bar{a}_2/a_1(\gamma)$.

Proof: See Appendix B.2.

Figure 4 displays the configuration of parameters for the equilibrium full-tax (zone I) and partial-tax (zone II) cases.

Figure 4: **Equilibrium taxation with asymmetric market sizes.** Case: $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$, $c_A = c_B = c_P$, and $a_2 > a_1$.



Three explanations are in order. Firstly, for a given asymmetry, upstream tax rates decrease, and dowstream tax rates increase, in γ, for the same reasons as in the symmetric case. Secondly, asymmetries in market sizes turn into asymmetries in tax rates. Specifically, $t_2 > t_1$, i.e., province 2 biases taxes to downstream transactions (where the tax base is relatively larger); while $\tau_1 > \tau_2$, i.e., province 1 biases taxes to upstream transactions. The rates $\{t_2, \tau_1, \tau_2\}$ increase in a_2. Also, for large values of a_2, province 1 drops down downstream taxes (t_1) and concentrates only on upstream taxation (again, to take advantage of the larger tax base in the retail market 2). In our case, this happens for intermediate-to-high product differentiation, as we rule out cases with $t_1 = 0$ and high γ because of negative profitability in the downstream market $1A$. Thirdly, asymmetries in upstream taxes tend to disappear as $\gamma \to \infty$ because upstream tax rates tend to zero; however, downstream rates remain asymmetric as provinces collect retail taxes in differently sized markets.

In the second case, we assume asymmetries in retail costs by province.

Proposition 3 *Regional asymmetries in retail costs. Assume symmetric market sizes ($a_1 = a_2 = a$), symmetric producer costs ($c_A = c_B = c_P$) and symmetric retailer costs by region ($c_{1A} = c_{1B} = c_1, = c_{2A} = c_{2B} = c_2$), but allow asymmetries between regional retail costs $c_1 \geq c_2$. There are two functions $\underline{c}_1/c_2(\gamma)$ and $\bar{c}_1/c_2(\gamma)$, continuous and increasing, that intersect at $\hat{\gamma}_{c2}$. If $\gamma < \hat{\gamma}_{c2}$, equilibrium taxes are such that $\{t_2 > t_1 > 0; \tau_1 > \tau_2 > 0\}$ if $c_1/c_2 \leq \underline{c}_1/c_2(\gamma)$ and $\{t_1 = 0; t_2 > 0; \tau_1 > \tau_2 > 0\}$ if $\underline{c}_1/c_2(\gamma) < c_1/c_2 \leq \bar{c}_2/c_1(\gamma)$. If $\gamma \geq \hat{\gamma}_{c2}$, equilibrium taxes are such that $\{t_2 > t_1 > 0; \tau_1 > \tau_2 > 0\}$ if $c_1/c_2 \leq \bar{c}_1/c_2(\gamma)$.*

Proof: See Appendix B.3.

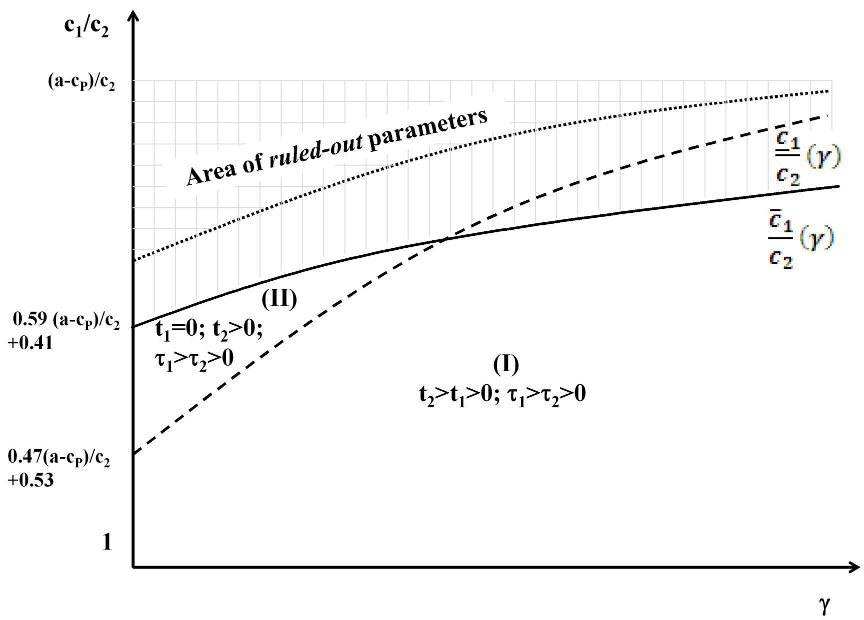
Figure 5 displays the configuration of parameters for the equilibrium full-tax (zone I) and partial-tax (zone II) cases.

Results from Proposition 3 mirror those of Proposition 2 as $c_1 > c_2$ is qualitatively similar to $a_2 > a_1$ in terms of net market value ($a_1 - c_1 - c_P$ vs $a_2 - c_2 - c_P$). Therefore, upstream rates decrease and downstream rates increase in γ; t_1 decreases and t_2 increase in c_1/c_2 (substitution effect between t_1 and c_1). Both rates τ_1 and τ_2 decrease in c_1 (with $\tau_2 < \tau_1$ because t_2 increases and t_1 decreases, eventually to 0).

In the third case, we assume asymmetries in producer costs.

Proposition 4 *Asymmetries in producer costs. Assume symmetric market sizes ($a_1 = a_2 = a$) and retail cost structures ($c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$), and let $c_A > c_B$. There is*

Figure 5: **Equilibrium taxation with asymmetric regional costs by region.** Case: $a_1 = a_2 = a$, $c_A = c_B = c_P$, $c_{1A} = c_{1B} = c_1$, $c_{2A} = c_{2B} = c_2$, and $c_1 \geq c_2$.



a function $\bar{c}_A/c_B(\gamma)$*, continuous and decreasing in* γ*, such that* $\{t_2 > t_1 > 0; \tau_1 > \tau_2 > 0\}$ *for* $c_A/c_B < \underline{c}_A/c_B(\gamma)$.

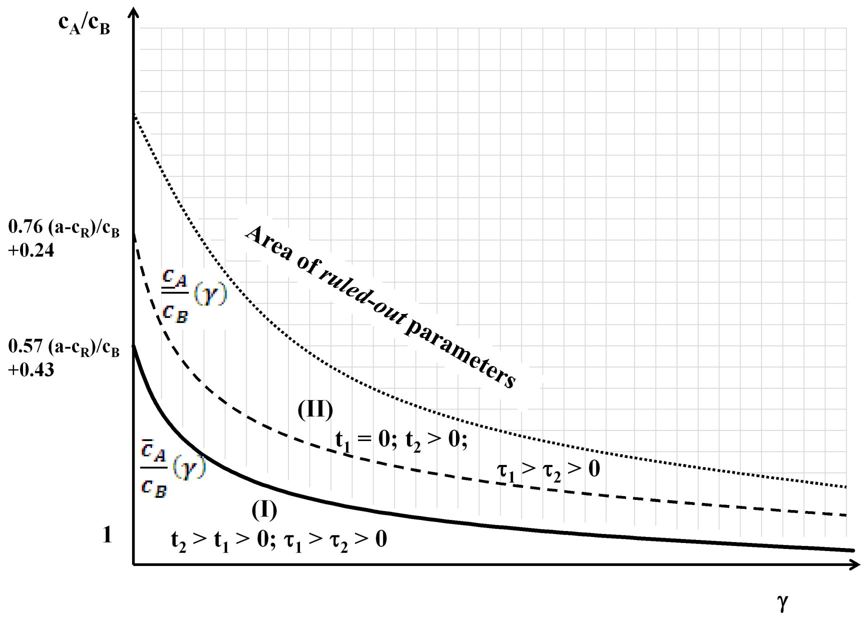
Proof: See Appendix B.4.

Figure 6 displays the configuration of parameters for the relevant equilibrium zone (which corresponds to the zone I of full taxation).[10]

We highlight two results. First, for a certain degree of product differentiation, the higher the relative cost of product A the relative market size of product A is reduced. This implies that government in province 1 reduces both τ_1 (the most affected rate) and t_1. Government 2 adjusts upstream tax rate downwards (a strategic complement effect to τ_1) and increases downstream rate (a strategic substitute effect to τ_1). Second, for a given

[10] Even though there is a feasible zone with partial tax rates (specifically, $t_1 = 0$), the constraint on non-negative markups is binding within this zone.

Figure 6: **Equilibrium taxation with asymmetric producer costs.** Case: $a_1 = a_2 = a$, $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$, and $c_A > c_B$.



asymmetry in producer costs, upstream rates decrease, downstream rate t_2 increases in product homogeneity, while t_1 increases for low and intermediate values of γ, which is a standard result.[11]

In the fourth case, we assume asymmetries in retail costs by product.

Proposition 5 *Asymmetries in retail costs by products. Assume symmetric market sizes ($a_1 = a_2 = a$), symmetric producer costs ($c_A = c_B = c_P$) and symmetric retailer costs by product ($c_{1A} = c_{2A} = c_{RA}, = c_{1B} = c_{2B} = c_{RB}$), but allow asymmetries between retail costs*

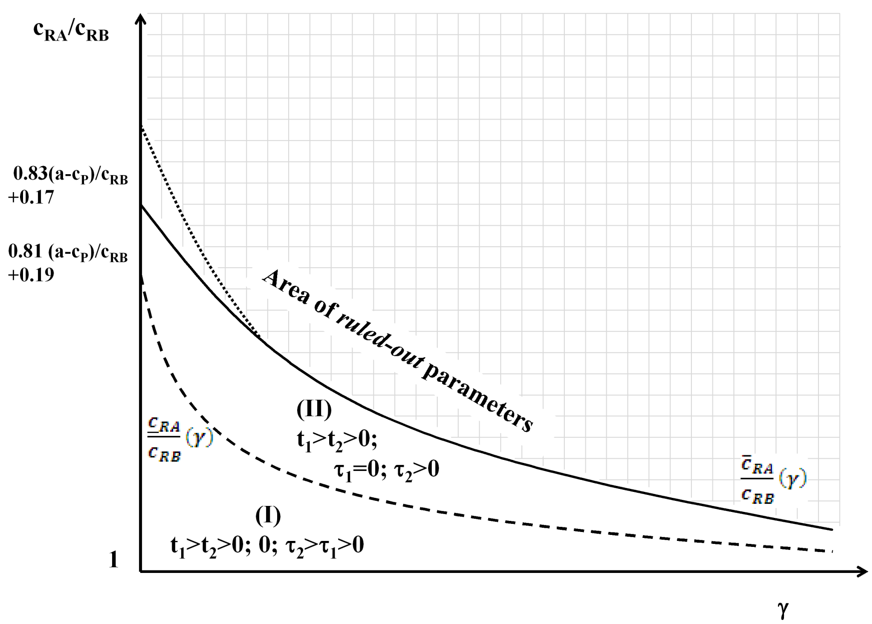
[11] We showed in the proof of Proposition 4 that τ_1 increases in c_A for $\gamma > \gamma_{AB} \simeq 10$. Also, for some intermediate value of product differentiation, t_1 decreases in γ. Although it may seem at first unclear, this result is a consequence of a strong reduction in t_1 due to the increase in c_A, which overcomes the standard effect of an increase in t_1 with γ. Accordingly, the government in province 1 switches taxes from t_1 to τ_1.

by products $c_{RA} \geq c_{RB}$. There are two functions $\underline{c}_{RA}/c_{RB}(\gamma)$ and $\bar{c}_{RA}/c_{RB}(\gamma)$, continuous and decreasing. Equilibrium taxes are such that $\{t_1 > t_2 > 0; \tau_2 > \tau_1 > 0\}$ if $c_{RA}/c_{RB} \leq \underline{c}_{RA}/c_{RB}(\gamma)$ and $\{t_1 > t_2 > 0; \tau_1 = 0; \tau_2 > 0\}$ if $\underline{c}_{RA}/c_{RB}(\gamma) < c_{RA}/c_{RB} \leq \bar{c}_{RA}/c_{RB}(\gamma)$

Proof: See Appendix B.5.

Figure 7 displays the configuration of parameters for the equilibrium full-tax (zone I) and partial-tax (zone II) cases.

Figure 7: **Equilibrium taxation with asymmetric retail costs by product.** Case: $a_1 = a_2 = a$, $c_A = c_B = c_P$, $c_{1A} = c_{2A} = c_{RA}$, $c_{1B} = c_{2B} = c_{RB}$, and $c_{RA} \geq c_{RB}$



As in the previous case, the link of tax rates with γ is straightforward. Second, a relative increase in the retail cost c_{RA}/c_{RB} reduces the market size of retail markets 1A and 2A (and also the product market A), inducing t_1, t_2 and τ_1 downwards, and τ_2 upwards; this way province 2 reacts strategically by taxing more heavily product B in the production source. Third, as the asymmetry becomes intense, government in province 1 foregoes upstream taxation to compensate downstream costs ($\tau_1 = 0$).

We conclude that provided that rates are non-negative (i.e., subsidies are ruled out) and products display a certain level of heterogeneity (γ is finite), either all rates are positive or at most one rate is set to zero. From Propositions 2 to 5 no two cases with zero-tax rate overlap: if $a_2 >> a_1$ (so that $t_1 = 0$) both τ_1 and τ_2 adjust increasingly in a_2; which cannot be met at the same time with c_{RA}/c_{RB} too high (low) for $\tau_2 = 0$ ($\tau_1 = 0$). Finally, when products are homogeneous ($\gamma \to \infty$) only downstream rates are positive, while upstream rates converge to zero. This is summarized in the next proposition.

Proposition 6 *Assume both governments choose tax rates in order to maximize revenues (Leviathan solution). If γ is finite and parameters are such that an equilibrium exists, tax rates are $(t_1^*, \tau_1^*, t_2^*, \tau_2^*)$ either all positive or only one of them equal to zero. If $\gamma \to \infty$, both upstream rates are equal to zero.*

4.2 Upstream / downstream taxation

Assume governments collect upstream taxes. Setting $t_1 = 0$ and $t_2 = 0$, and adapting conditions (9) and (11) to upstream rates, the solution is

$$\tau_1 = \frac{2\mu(\Gamma_{1A} + \Gamma_{2A}) + \kappa(\Gamma_{1B} + \Gamma_{2B})}{2(4\mu^2 - \kappa^2)} \quad ; \quad \tau_2 = \frac{\kappa(\Gamma_{1A} + \Gamma_{2A}) + 2\mu(\Gamma_{1B} + \Gamma_{2B})}{2(4\mu^2 - \kappa^2)}$$

On the other hand, if governments collect downstream taxes, setting $\tau_1 = 0$ and $\tau_2 = 0$, and adapting conditions (8) and (10) to downstream rates, the solution is

$$t_1 = \frac{2\alpha(\Gamma_{1A} + \Gamma_{1B}) + \beta(\Gamma_{2A} + \Gamma_{2B})}{2(4\alpha^2 - \beta^2)} \quad ; \quad t_2 = \frac{\beta(\Gamma_{1A} + \Gamma_{1B}) + 2\alpha(\Gamma_{2A} + \Gamma_{2B})}{2(4\alpha^2 - \beta^2)}$$

Proposition 7 *Revenue with full rates is higher than revenue with upstream / downstream taxes if γ is finite, and equal to revenue with downstream taxes if goods are homogeneous.*

Proof: Proposition 6 states that the full solution involves at least one more active rate than the solution with upstream or downstream rates. Only in the case of $\gamma \to \infty$ the full-rate solution converges to downstream taxation. *Q.E.D.*

Therefore, there is strong ground for using successive taxation when the objective of provincial governments is to collect taxes. A companion paper analyzes the robustness of this result to other objective functions, particularly welfare functions (subject to a given level of revenue needs).

4.3 Cooperative solution with full taxes

This section explores the relationship between competitive taxation and coordinated taxation, and focuses on whether uncoordinated taxation involves over-taxation (a standard result in the literature of competitive taxation) and on whether all rates are active (or if coordination suggests using only upstream or downstream rates). Assume that provinces 1 and 2 coordinate their selection of tax rates. The first-order conditions corresponding to the joint maximization of $R_1 + R_2$ are

$$t_1 \quad : \quad \Gamma_{1A} + \Gamma_{1B} - 4\alpha t_1 + 4\beta t_2 - 2\psi\tau_1 - 2\psi\tau_2 = 0 \tag{14}$$

$$\tau_1 \quad : \quad \Gamma_{1A} + \Gamma_{2A} - 4\mu\tau_1 + 4\kappa\tau_2 - 2\psi t_1 - 2\psi t_2 = 0 \tag{15}$$

$$t_2 \quad : \quad \Gamma_{2A} + \Gamma_{2B} - 4\alpha t_2 + 4\beta t_1 - 2\psi\tau_2 - 2\psi\tau_1 = 0 \tag{16}$$

$$\tau_2 \quad : \quad \Gamma_{2B} + \Gamma_{1B} - 4\mu\tau_2 + 4\kappa\tau_1 - 2\psi t_2 - 2\psi t_1 = 0 \tag{17}$$

First, working on these conditions, the solution is not unique, but rather involves a (multiple) combination of tax rates such that

$$t_1^C + t_2^C + \tau_1^C + \tau_2^C = \frac{\Gamma_{1A} + \Gamma_{1B} + \Gamma_{2A} + \Gamma_{2B}}{4\psi} \tag{18}$$

$$t_1^C - t_2^C = \frac{\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B}}{4(\alpha + \beta)} \tag{19}$$

$$\tau_1^C - \tau_2^C = \frac{\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B}}{4(\mu + \kappa)} \tag{20}$$

Notice that the solution from conditions (18)-(20) could draw two, three or four non-negative tax rates. The case with two non-zero rates could arise in very particular cases, such as large Γ_{1A} and small $\Gamma_{1B}, \Gamma_{2A}, \Gamma_{2B}$, in which case it is optimal to tax in province 1, or large market 1 (both Γ_{1A} and Γ_{1B}), in which case it is optimal to downstream tax in province 1 (t_1) and upstream tax in province 2 (τ_2).

Second, Proposition 8 provides a straightforward result regarding over-taxation:

Proposition 8 *The equilibrium tax rates imply over-taxation if there is a minimum degree of product differentiation, and the level of cooperative taxation if products are homogeneous.*

Proof: Consider the full solution with positive rates from equations (38)-(41). The sum of all four rates is equal to

$$\frac{2(\Gamma_{1A} + \Gamma_{1B} + \Gamma_{2A} + \Gamma_{2B})(2\alpha - \beta + 2\mu - \kappa - 3\psi)}{4(2\alpha - \beta)(2\mu - \kappa) - 9\psi^2}$$

Comparing this condition with (18), it is easy to verify that

$$H(\gamma) = \frac{2(2\alpha - \beta - \kappa + 2\mu - 3\psi)}{4(2\alpha - \beta)(2\mu - \kappa) - 9\psi^2} > \frac{1}{4\psi}$$

for finite γ and $H(\infty) \to 1 = \frac{1}{\psi(\infty)}$. *Q.E.D.*

When the objective of provincial government is maximize tax revenues, the choice of tax rates in equilibrium leaves them with lower level of tax revenues. Only when products are homogeneous ($\gamma \to \infty$) the competitive solution equals the cooperative one.

The replacement of Γ_{1A} to Γ_{2B} by their corresponding definitions into (18) generates the following condition

$$t_1^C + t_2^C + \tau_1^C + \tau_2^C = \frac{1}{4}\left[2(a_1 + a_2) - 2(c_A + c_B) - (c_{1A} + c_{1B} + c_{2A} + c_{2B})\right] \qquad (21)$$

which states that the sum of tax rates under the cooperative solution is independent of the degree of product differentiation (γ). On the other hand, equilibrium taxation depends on the level of product differentiation. When products are extremely heterogeneous ($\gamma = 0$) the degree of over-taxation is small. But the difference between cooperative tax revenue and equilibrium joint tax revenue increases as products are less heterogeneous.

5 Conclusions and future research

This paper analyzes multi-stage taxation by provinces in a federal country. In order to do so, we set up a two-good, two-province, two-stage successive differentiated-product oligopoly model, where each producer is located in a province and sells its product through exclusive retailers located in both provinces. Retailers compete for consumers *a la* Bertrand with differentiated products. The producer-retailer setup allows provincial governments to raise taxes on both upstream and downstream links of the value chain. With this model, we study tax incidence and Leviathan non-cooperative taxation.

The tax incidence properties of the model are such that the effect of changes in upstream or downstream tax rates on producer and retailer prices depend crucially on the degree of product differentiation and therefore on the level of (downstream and upstream) market power.

In the full-tax solution we find that provinces raise both upstream and downstream taxes, but the mix between the two instruments depends on the degree of downstream competition and market asymmetries. When there is little downstream competition provinces set rates so that upstream transactions are more heavily taxed (double marginalization effect), but the tax mix shifts towards retail transactions as competition becomes more intense. In sufficiently asymmetric cases (on demand size or cost) one province may drop one tax rate. The retail rate is set to zero when demands, retail cost by region or producer costs are sufficiently different. On the other hand, the producer rate is set to zero when retail costs by product are sufficiently different. We compare the full-tax solution with specific cases (upstream and downstream taxation, and cooperative taxation). We find that multi-stage taxation dominates other forms of indirect taxation (either downstream or upstream) and implies over-taxation when compared with coordinated provincial taxation.

The paper is part of a research agenda which studies successive taxation. While the main focus was on revenue maximization, we left aside an important component of provincial objectives, which is local welfare. A companion paper analyzes this extension. Also, we focused the analysis on provincial governments within a country, but the main problem applies to national governments within a union. In particular, successive taxation was not considered so far in the analysis of origin vs destination principles.

This paper challenges the cascade effect on the grounds that, when (local) governments choose what link of the value chain to tax, this effect trades off against import or export of tax bases. This issue is very important within the tax harmonization literature and deserves future research.

Appendix

A Relevant parameters and no-tax equilibrium

From section 2.2.2 we find first-order conditions for p_A and p_B and solve for equilibrium prices. Constants terms (other than those multiplying tax rates) are:

$$\Phi_A = \delta\left(a_1 + a_2 - c_{1A} - c_{2A}\right) + \frac{\theta}{2}\left[c_{1B} + c_{2B} - c_{1A} - c_{2A}\right] + \omega c_A + \theta c_B \tag{22}$$

$$\Phi_B = \delta\left(a_1 + a_2 - c_{1B} - c_{2B}\right) + \frac{\theta}{2}\left[c_{1A} + c_{2A} - c_{1B} - c_{2B}\right] + \theta c_A + \omega c_B \tag{23}$$

Replacing p_A and p_A into p_{1A} to p_{2B} we get retailers prices. Constants terms (other than those multiplying tax rates) are:

$$\Psi_{1A} = \frac{a_1}{(2+\gamma)} + \frac{(1+\gamma)\left[2(1+\gamma)(\Phi_A + c_{1A}) + \gamma(\Phi_B + c_{1B})\right]}{(2+\gamma)(2+3\gamma)} \tag{24}$$

$$\Psi_{1B} = \frac{a_1}{(2+\gamma)} + \frac{(1+\gamma)\left[2(1+\gamma)(\Phi_B + c_{1B}) + \gamma(\Phi_A + c_{1A})\right]}{(2+\gamma)(2+3\gamma)} \tag{25}$$

$$\Psi_{2A} = \frac{a_2}{(2+\gamma)} + \frac{(1+\gamma)\left[2(1+\gamma)(\Phi_A + c_{2A}) + \gamma(\Phi_B + c_{2B})\right]}{(2+\gamma)(2+3\gamma)} \tag{26}$$

$$\Psi_{2B} = \frac{a_2}{(2+\gamma)} + \frac{(1+\gamma)\left[2(1+\gamma)(\Phi_B + c_{2B}) + \gamma(\Phi_A + c_{2A})\right]}{(2+\gamma)(2+3\gamma)} \tag{27}$$

Finally, replacing prices into x_{1A} to x_{2B} we get quantities. Constants terms (other than those multiplying tax rates) are:

$$\begin{aligned}\Gamma_{1A} = {} & \alpha(a_1 - c_{1A}) - \beta\left(a_2 - c_{2A}\right) - \frac{(1+\gamma)}{(2+\gamma)}\left(\omega c_A + \theta c_B\right) + \frac{(1+\gamma)^2}{2(2+\gamma)^2}\frac{\theta}{\beta}(c_B - c_A) \\ & + \frac{\gamma(1+\gamma)^2}{4(2+\gamma)(2+3\gamma)}\left[(2+\omega)\left(c_{1B} - c_{1A}\right) - (2-\omega)\left(c_{2B} - c_{2A}\right)\right]\end{aligned} \tag{28}$$

$$\begin{aligned}\Gamma_{1B} = {} & \alpha(a_1 - c_{1B}) - \beta\left(a_2 - c_{2B}\right) - \frac{(1+\gamma)}{(2+\gamma)}\left(\omega c_A + \theta c_B\right) + \frac{(1+\gamma)^2}{2(2+\gamma)^2}\frac{\theta}{\beta}(c_B - c_A) \\ & + \frac{\gamma(1+\gamma)^2}{4(2+\gamma)(2+3\gamma)}\left[(2+\omega)\left(c_{1A} - c_{1B}\right) - (2-\omega)\left(c_{2A} - c_{2B}\right)\right]\end{aligned} \tag{29}$$

$$\begin{aligned}\Gamma_{2A} = {} & \alpha(a_2 - c_{2A}) - \beta\left(a_1 - c_{1A}\right) - \frac{(1+\gamma)}{(2+\gamma)}\left(\omega c_B + \theta c_A\right) + \frac{(1+\gamma)^2}{2(2+\gamma)^2}\frac{\theta}{\beta}(c_A - c_B) \\ & + \frac{\gamma(1+\gamma)^2}{4(2+\gamma)(2+3\gamma)}\left[(2+\omega)\left(c_{2B} - c_{2A}\right) - (2-\omega)\left(c_{1B} - c_{1A}\right)\right]\end{aligned} \tag{30}$$

$$\Gamma_{2B} = \alpha(a_2 - c_{2B}) - \beta\left(a_1 - c_{1B}\right) - \frac{(1+\gamma)}{(2+\gamma)}\left(\omega c_B + \theta c_A\right) + \frac{(1+\gamma)^2}{2(2+\gamma)^2}\frac{\theta}{\beta}(c_A - c_B)$$

$$+\frac{\gamma(1+\gamma)^2}{4(2+\gamma)(2+3\gamma)}\left[(2+\omega)\left(c_{2A}-c_{2B}\right)-(2-\omega)\left(c_{1A}-c_{1B}\right)\right] \tag{31}$$

where we make use of the following definitions:

$$\alpha = \frac{3(1+\gamma)(2+3\gamma)+2\gamma(1+\gamma)^2}{2\left[2(2+3\gamma)+\gamma(1+\gamma)\right](2+\gamma)} \quad , \quad \beta = \frac{(1+\gamma)(2+3\gamma)}{2\left[2(2+3\gamma)+\gamma(1+\gamma)\right](2+\gamma)} \tag{32}$$

$$\Delta = \frac{1}{\left[2(2+3\gamma)+3\gamma(1+\gamma)\right]\left[2(2+3\gamma)+\gamma(1+\gamma)\right]} \quad , \quad \theta = \gamma(1+\gamma)\lambda \tag{33}$$

$$\omega = 2[(2+3\gamma)+\gamma(1+\gamma)]^2\Delta \quad , \quad \delta = \frac{(2+3\gamma)}{2[2(2+3\gamma)+\gamma(1+\gamma)]} \tag{34}$$

$$\lambda = \left[(2+3\gamma)+\gamma(1+\gamma)\right]\Delta \quad , \quad \rho = \frac{(1+\gamma)(2\omega(1+\gamma)+\gamma\theta)}{(2+\gamma)(2+3\gamma)} \tag{35}$$

$$\mu = \frac{(1+\gamma)}{4(2+\gamma)}\frac{\omega}{\delta}+\frac{\gamma(1+\gamma)^2}{(2+\gamma)}\lambda \quad , \quad \kappa = \frac{\gamma(1+\gamma)^2}{2(2+\gamma)(2+3\gamma)}\omega \tag{36}$$

$$\sigma = \frac{(1+\gamma)(2\theta(1+\gamma)+\gamma\omega)}{(2+\gamma)(2+3\gamma)} \quad , \quad \psi = \frac{(1+\gamma)[(2+3\gamma)+\gamma(1+\gamma)]}{(2+\gamma)\left[2(2+3\gamma)+\gamma(1+\gamma)\right]} \tag{37}$$

Notice that (Φ_A, Φ_B) are the no-tax equilibrium upstream prices, $(\Psi_{1A}, \Psi_{1B}, \Psi_{2A}, \Psi_{2B})$ are the no-tax equilibrium downstream prices, and $(\Gamma_{1A}, \Gamma_{1B}, \Gamma_{2A}, \Gamma_{2B})$ are the no-tax equilibrium quantities. We assume parameters such that the no-tax solution is interior. In the following proofs, parameters that do not satisfy this assumption will be referred to as *ruled-out parameters*.

B Proof of Propositions 1 to 3

As a preface to the proof of propositions 1 to 3, the interior solution to (8)-(9) can be written as

$$t_1^* = \frac{\begin{array}{l}\left(\Gamma_{1A}+\Gamma_{1B}\right)\left(64\alpha\mu^2-8\psi^2\kappa-20\psi^2\mu-16\alpha\kappa^2\right)\\ -\left(\Gamma_{2A}+\Gamma_{2B}\right)\left(8\beta\kappa^2-10\psi^2\kappa-16\psi^2\mu-32\beta\mu^2\right)\\ -\left(\Gamma_{1A}+\Gamma_{2A}\right)\left(8\psi\alpha\kappa+8\psi\beta\kappa+32\psi\alpha\mu+8\psi\beta\mu-6\psi^3\right)\\ -\left(\Gamma_{1B}+\Gamma_{2B}\right)\left(16\psi\alpha\kappa+4\psi\beta\kappa+16\psi\alpha\mu+16\psi\beta\mu+3\psi^3\right)\end{array}}{9\psi^4-8\psi^2(5\beta\kappa+8\beta\mu+8\alpha\kappa+20\alpha\mu)+16(4\mu^2-\kappa^2)(4\alpha^2-\beta^2)} \tag{38}$$

$$\tau_1^* = \frac{\begin{array}{l}\left(\Gamma_{1A}+\Gamma_{1B}\right)\left(6\psi^3-32\psi\alpha\mu-8\psi\beta\kappa-8\psi\alpha\kappa-8\psi\beta\mu\right)\\ -\left(\Gamma_{2A}+\Gamma_{2B}\right)\left(3\psi^3+16\psi\alpha\kappa+16\psi\beta\mu+16\psi\alpha\mu+4\psi\beta\kappa\right)\\ +\left(\Gamma_{1A}+\Gamma_{2A}\right)\left(64\alpha^2\mu-20\psi^2\alpha-6\psi^2\beta-16\beta^2\mu\right)\\ +\left(\Gamma_{1B}+\Gamma_{2B}\right)\left(16\psi^2\alpha+8\psi^2\beta+32\alpha^2\kappa-8\beta^2\kappa\right)\end{array}}{9\psi^4-8\psi^2(5\beta\kappa+8\beta\mu+8\alpha\kappa+20\alpha\mu)+16(4\mu^2-\kappa^2)(4\alpha^2-\beta^2)} \tag{39}$$

$$t_2^* = \frac{\begin{array}{l}(\Gamma_{2A}+\Gamma_{2B})\left(64\alpha\mu^2-8\psi^2\kappa-20\psi^2\mu-16\alpha\kappa^2\right)\\ -(\Gamma_{1A}+\Gamma_{1B})\left(8\beta\kappa^2-10\psi^2\kappa-16\psi^2\mu-32\beta\mu^2\right)\\ -(\Gamma_{1A}+\Gamma_{2A})\left(16\psi\alpha\kappa+4\psi\beta\kappa+16\psi\alpha\mu+16\psi\beta\mu+3\psi^3\right)\\ -(\Gamma_{1B}+\Gamma_{2B})\left(8\psi\alpha\kappa+8\psi\beta\kappa+32\psi\alpha\mu+8\psi\beta\mu-6\psi^3\right)\end{array}}{9\psi^4-8\psi^2(5\beta\kappa+8\beta\mu+8\alpha\kappa+20\alpha\mu)+16(4\mu^2-\kappa^2)(4\alpha^2-\beta^2)} \tag{40}$$

$$\tau_2^* = \frac{\begin{array}{l}(\Gamma_{2A}+\Gamma_{2B})\left(6\psi^3-32\psi\alpha\mu-8\psi\beta\kappa-8\psi\alpha\kappa-8\psi\beta\mu\right)\\ -(\Gamma_{1A}+\Gamma_{1B})\left(3\psi^3+16\psi\alpha\kappa+16\psi\beta\mu+16\psi\alpha\mu+4\psi\beta\kappa\right)\\ +(\Gamma_{1B}+\Gamma_{2B})\left(64\alpha^2\mu-20\psi^2\alpha-6\psi^2\beta-16\beta^2\mu\right)\\ +(\Gamma_{1A}+\Gamma_{2A})\left(16\psi^2\alpha+8\psi^2\beta+32\alpha^2\kappa-8\beta^2\kappa\right)\end{array}}{9\psi^4-8\psi^2(5\beta\kappa+8\beta\mu+8\alpha\kappa+20\alpha\mu)+16(4\mu^2-\kappa^2)(4\alpha^2-\beta^2)} \tag{41}$$

where Γ_{1A} to Γ_{2B} are defined in (28)-(31). Alternatively, replacing Γ_{1A} to Γ_{2B} into (38)-(41) the tax rates can be written as a function of main parameters $(a_1, a_2, c_A, c_B, c_{1A}, c_{1B}, c_{2A}, c_{2B}, \gamma)$:

$$t_1 = w_{a1}^{t1}a_1 + w_{a2}^{t1}a_2 + w_{c1A}^{t1}c_{1A} + w_{c1B}^{t1}c_{1B} + w_{c2A}^{t1}c_{2A} + w_{c2B}^{t1}c_{2B} + w_{cA}^{t1}c_A + w_{cB}^{t1}c_B \tag{42}$$

$$\tau_1 = w_{a1}^{\tau1}a_1 + w_{a2}^{\tau1}a_2 + w_{c1A}^{\tau1}c_{1A} + w_{c1B}^{\tau1}c_{1B} + w_{c2A}^{\tau1}c_{2A} + w_{c2B}^{\tau1}c_{2B} + w_{cA}^{\tau1}c_A + w_{cB}^{\tau1}c_B \tag{43}$$

$$t_2 = w_{a1}^{t2}a_1 + w_{a2}^{t2}a_2 + w_{c1A}^{t2}c_{1A} + w_{c1B}^{t2}c_{1B} + w_{c2A}^{t2}c_{2A} + w_{c2B}^{t2}c_{2B} + w_{cA}^{t2}c_A + w_{cB}^{t2}c_B \tag{44}$$

$$\tau_2 = w_{a1}^{\tau2}a_1 + w_{a2}^{\tau2}a_2 + w_{c1A}^{\tau2}c_{1A} + w_{c1B}^{\tau2}c_{1B} + w_{c2A}^{\tau2}c_{2A} + w_{c2B}^{\tau2}c_{2B} + w_{cA}^{\tau2}c_A + w_{cB}^{\tau2}c_B \tag{45}$$

When parameters are such that (38) is negative, the solution is $t_1^* = 0$ and

$$\tau_1^* = \frac{(4\alpha\kappa+\psi^2)(\Gamma_{1B}+\Gamma_{2B})+2(4\alpha\mu-\psi^2)(\Gamma_{1A}+\Gamma_{2A})-2\psi(\mu+\kappa)(\Gamma_{2A}+\Gamma_{2B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{46}$$

$$t_2^* = \frac{-\psi(4\mu+\kappa)(\Gamma_{1B}+\Gamma_{2B})-2\psi(\mu+\kappa)(\Gamma_{1A}+\Gamma_{2A})+2(4\mu^2-\kappa^2)(\Gamma_{2A}+\Gamma_{2B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{47}$$

$$\tau_2^* = \frac{(16\alpha\mu-\psi^2)(\Gamma_{1B}+\Gamma_{2B})+2(4\alpha\kappa+\psi^2)(\Gamma_{1A}+\Gamma_{2A})-2\psi(4\mu+\kappa)(\Gamma_{2A}+\Gamma_{2B})}{4(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{48}$$

When parameters are such that (40) is negative, the solution is $t_2^* = 0$ and

$$t_1^* = \frac{-\psi(4\mu+\kappa)(\Gamma_{1A}+\Gamma_{2A})-2\psi(\mu+\kappa)(\Gamma_{1B}+\Gamma_{2B})+2(4\mu^2-\kappa^2)(\Gamma_{1A}+\Gamma_{1B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{49}$$

$$\tau_1^* = \frac{(16\alpha\mu-\psi^2)(\Gamma_{1A}+\Gamma_{2A})+2(4\alpha\kappa+\psi^2)(\Gamma_{1B}+\Gamma_{2B})-2\psi(4\mu+\kappa)(\Gamma_{1A}+\Gamma_{1B})}{4(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{50}$$

$$\tau_2^* = \frac{(4\alpha\kappa+\psi^2)(\Gamma_{1A}+\Gamma_{2A})+2(4\alpha\mu-\psi^2)(\Gamma_{1B}+\Gamma_{2B})-2\psi(\mu+\kappa)(\Gamma_{1A}+\Gamma_{1B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{51}$$

When parameters are such that (39) is negative, the solution is $\tau_1^* = 0$ and

$$t_1^* = \frac{2\psi(\alpha+\beta)(\Gamma_{1B}+\Gamma_{2B}) + 2(4\alpha\mu-\psi^2)(\Gamma_{1A}+\Gamma_{1B}) + (2\beta\mu-\psi^2)(\Gamma_{2A}+\Gamma_{2B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{52}$$

$$t_2^* = \frac{-2\psi(4\alpha+\beta)(\Gamma_{1B}+\Gamma_{2B}) + 2(4\beta\mu-\psi^2)(\Gamma_{1A}+\Gamma_{1B}) - (16\alpha\mu-\psi^2)(\Gamma_{2A}+\Gamma_{2B})}{4(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{53}$$

$$\tau_2^* = \frac{-2(4\alpha^2-\beta^2)(\Gamma_{1B}+\Gamma_{2B}) + 2\psi(\alpha+\beta)(\Gamma_{1A}+\Gamma_{1B}) + \psi(4\alpha+\beta)(\Gamma_{2A}+\Gamma_{2B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{54}$$

When parameters are such that (41) is negative, the solution is $\tau_2^* = 0$ and

$$t_1^* = \frac{2\psi(4\alpha+\beta)(\Gamma_{1A}+\Gamma_{2A}) + (16\alpha\mu-\psi^2)(\Gamma_{1A}+\Gamma_{1B}) + 2(4\beta\mu+\psi^2)(\Gamma_{2A}+\Gamma_{2B})}{4(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{55}$$

$$\tau_1^* = \frac{-2(4\alpha^2-\beta^2)(\Gamma_{1A}+\Gamma_{2A}) + (4\alpha+\beta)(\Gamma_{1A}+\Gamma_{1B}) + 2\psi(\alpha+\beta)(\Gamma_{2A}+\Gamma_{2B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{56}$$

$$t_2^* = \frac{-2\psi(\alpha+\beta)(\Gamma_{1A}+\Gamma_{2A}) - (4\beta\mu+\psi^2)(\Gamma_{1A}+\Gamma_{1B}) - 2(4\alpha\mu-\psi^2)(\Gamma_{2A}+\Gamma_{2B})}{2(4\alpha(4\mu^2-\kappa^2)-\psi^2(5\mu+2\kappa))} \tag{57}$$

B.1 Proof of proposition 1

If demand and costs are symmetric ($a_1 = a_2 = a$, $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$, $c_A = c_B = c_P$), then the functions Γ_{1A} to Γ_{2B} defined in (28)-(31) simplify to

$$\Gamma_{1A} = \Gamma_{1B} = \Gamma_{2A} = \Gamma_{2B} = \Gamma = \psi(a - c_R - c_P) \tag{58}$$

Replacing (58) into (38)-(41), tax rates simplify to (12)-(13). *Q.E.D.*

B.2 Proof of Proposition 2

Assume $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$, $c_A = c_B = c_P$, and asymmetric demands with $a_2 \geq a_1$. The functions Γ_{1A} to Γ_{2B} defined in (28)-(31) simplify to

$$\Gamma_{1A} = \Gamma_{1B} = \alpha a_1 - \beta a_2 - \psi(c_R + c_P) \tag{59}$$

$$\Gamma_{2A} = \Gamma_{2B} = \alpha a_2 - \beta a_1 - \psi(c_R + c_P) \tag{60}$$

Replacing these values into (38)-(41), rates under full taxation can be written as functions of a_1, a_2 and $c_R + c_P$. Equations (42)-(45) simplify to

$$\begin{aligned}
t_1 &= w^t_{a1}a_1 - w^t_{a2}a_2 - w^t_{RP}(c_R + c_P) \\
\tau_1 &= w^\tau_{a1}a_1 + w^\tau_{a2}a_2 - w^\tau_{RP}(c_R + c_P) \\
t_2 &= -w^t_{a2}a_1 + w^t_{a1}a_2 - w^t_{RP}(c_R + c_P) \\
\tau_2 &= w^\tau_{a2}a_1 + w^\tau_{a1}a_2 - w^\tau_{RP}(c_R + c_P)
\end{aligned}$$

where $(w^t_{a1}, w^t_{a2}, w^t_{RP}, w^\tau_{a1}, w^\tau_{a2}, w^\tau_{RP})$ are positive-value functions of γ.

First, notice that the rate t_1 decreases in a_2 and may become negative as a_2 grows large. In the full tax solution, given that t_2 increases in a_2, it becomes clear that $t_2 > t_1$ for all γ in the asymmetric-demand case when $a_2 > a_1$. Also, τ_1 and τ_2 grow with a_2, and it can be checked that $w^\tau_{a1}(\gamma) > w^\tau_{a2}(\gamma)$ and then $\tau_1 > \tau_2$ for all γ.

For high values of a_2, $t_1 = 0$ and the solution corresponds to partial taxation. The threshold for $t_1 = 0$ is obtained as follows. Let $\Upsilon_I = w^t_{RP}/w^t_{a2}$ and $\Omega_I = w^t_{a1}/w^t_{a2}$, and take $(c_R + c_P)/a_1$ as given. The cutoff value of a_2/a_1 for $t_1 = 0$ can be written as

$$\frac{a_2}{a_1}(\gamma) = \Omega_I(\gamma) - \Upsilon_I(\gamma)\frac{(c_R + c_P)}{a_1} \tag{61}$$

Equation (61) is depicted in Figure 4 (dashed line). It is such that $\Upsilon_I(0) = 54/61 \simeq 0.89$, $\Omega_I(0) = 115/61 \simeq 1.89$, and $\frac{a_2}{a_1}(\gamma)$ increases as $\gamma \to \infty$. (This function is always below the dotted line, that delimitates the ruled-out parameters area.)

When parameters are such that $\frac{a_2}{a_1} > \frac{a_2}{a_1}(\gamma)$, and provided that there is a solution (see below), $t_1 = 0$ and the other three rates can be obtained from (46)-(48), as follows

$$\begin{aligned}
\tau_1 &= w^{\tau 1}_{a1}a_1 + w^{\tau 1}_{a2}a_2 - w^{\tau 1}_{RP}(c_R + c_P) \\
t_2 &= -w^{t2}_{a1}a_1 + w^{t2}_{a2}a_2 - w^{t2}_{RP}(c_R + c_P) \\
\tau_2 &= w^{\tau 2}_{a1}a_1 + w^{\tau 2}_{a2}a_2 - w^{\tau 2}_{RP}(c_R + c_P)
\end{aligned}$$

where $(w^{t2}_{a1}, w^{t2}_{a2}, w^{t2}_{RP}, w^{\tau 1}_{a1}, w^{\tau 1}_{a2}, w^{\tau 1}_{RP}, w^{\tau 2}_{a1}, w^{\tau 2}_{a2}, w^{\tau 2}_{RP})$ are positive-value functions of γ. Again, it can be checked that $w^{\tau 2}_{a1}(\gamma) > w^{\tau 2}_{a2}(\gamma)$ and then $\tau_1 > \tau_2$ for all γ.

Second, asymmetries in market sizes and tax rates may render negative markups. After examination of all cases, the relevant markup that may become negative is $m_{1A} = p_{1A} - p_A - t_1 - c_{1A}$. Intuitively, (i) the asymmetry is such that $a_2/a_1 \geq 1$ and the candidate markets that may become unprofitable are the retail markets in province 1, and (ii) the tax-mix in province 1 shifts away from retail (t_1) to wholesale (τ_1), making product A more costly.

Introducing the tax rates (the four positive tax rates in the full-tax case or $t_1 = 0$ and the remaining rates in the partial-tax case) into m_{1A}, we look for the threshold a_2/a_1 such the markup is zero:

$$\begin{aligned} \frac{\bar{a}_2}{a_1}(\gamma) &= \frac{\alpha_{I1}}{\beta_{I1}} - \frac{\psi_{I1}}{\beta_{I1}} \frac{(c_R + c_P)}{a_1} \quad \text{if } \frac{a_2}{a_1} \leq \frac{\underline{a}_2}{a_1}(\gamma) \\ &= \frac{\alpha_{I2}}{\beta_{I2}} - \frac{\psi_{I2}}{\beta_{I2}} \frac{(c_R + c_P)}{a_1} \quad \text{if } \frac{a_2}{a_1} > \frac{\underline{a}_2}{a_1}(\gamma) \end{aligned} \tag{62}$$

where

$$\begin{aligned} \alpha_{I1} &= \frac{1-\delta}{2+\gamma} - (1-\alpha-\delta) w_{a1}^t - (\delta-\beta) w_{a2}^t - (\omega-\rho) w_{a1}^\tau + (\sigma-\theta) w_{a2}^\tau \\ \beta_{I1} &= \frac{\delta}{2+\gamma} - (1-\alpha-\delta) w_{a2}^t - (\delta-\beta) w_{a1}^t + (\omega-\rho) w_{a2}^\tau - (\sigma-\theta) w_{a1}^\tau \\ \psi_{I1} &= \frac{1-2\delta}{2+\gamma} - (1-\psi-2\delta) w_{RP}^t - (\omega-\rho-\sigma+\theta) w_{RP}^\tau \end{aligned}$$

$$\begin{aligned} \alpha_{I2} &= \frac{1-\delta}{2+\gamma} - (\delta-\beta) w_{a1}^{t2} - (\omega-\rho) w_{a1}^{\tau 1} + (\sigma-\theta) w_{a1}^{\tau 2} \\ \beta_{I2} &= \frac{\delta}{2+\gamma} - (\delta-\beta) w_{a2}^{t2} + (\omega-\rho) w_{a2}^{\tau 1} - (\sigma-\theta) w_{a2}^{\tau 2} \\ \psi_{I2} &= \frac{1-2\delta}{2+\gamma} + (\delta-\beta) w_{RP}^{t2} - (\omega-\rho) w_{RP}^{\tau 1} + (\sigma-\theta) w_{RP}^{\tau 2} \end{aligned}$$

Equation (62) is depicted in Figure 4 (solid line). In the region with $t_1 = 0$ it is such that $\frac{\alpha_{I2}}{\beta_{I2}}(0) = \frac{37}{15}$, $\frac{\psi_{I2}}{\beta_{I2}}(0) = \frac{22}{15}$, and $\frac{\bar{a}_2}{a_1}(\gamma)$ increases in γ but crossing $\frac{\underline{a}_2}{a_1}(\gamma)$ at $\gamma_{a2} \simeq 6.57$. At this point, it switches (continuously) to the function corresponding to the region with positive tax rates. *Q.E.D.*

B.3 Proof of proposition 3

Assume $a_1 = a_2 = a$, $c_A = c_B = c_P$, $c_{1A} = c_{1B} = c_1$, $c_{2A} = c_{2B} = c_2$ and let $c_1 \geq c_2$. The functions Γ_{1A} to Γ_{2B} defined in (28)-(31) simplify to

$$\Gamma_{1A} = \Gamma_{1B} = \psi(a - c_P) - \alpha c_1 + \beta c_2 \quad (63)$$

$$\Gamma_{2A} = \Gamma_{2B} = \psi(a - c_P) - \alpha c_2 + \beta c_1 \quad (64)$$

Replacing these values into (38)-(41), tax rates can be written as functions of $a - c_P$, c_1 and c_2. Equations (42)-(45) simplify to

$$\begin{aligned} t_1 &= w^t_{aP}(a - c_P) - w^t_{c1}c_1 + w^t_{c2}c_2 \\ \tau_1 &= w^\tau_{aP}(a - c_P) - w^\tau_{c1}c_1 - w^\tau_{c2}c_2 \\ t_2 &= w^t_{aP}(a - c_P) + w^t_{c2}c_1 - w^t_{c1}c_2 \\ \tau_2 &= w^\tau_{aP}(a - c_P) - w^\tau_{c2}c_1 - w^\tau_{c1}c_2 \end{aligned}$$

where $(w^t_{aP}, w^\tau_{aP}, w^t_{c1}, w^\tau_{c1}, w^t_{c2}, w^\tau_{c2})$ are positive-valued functions of γ.

First, notice that t_1, τ_1 and τ_2 can become negative if c_1 is large enough. For high values of c_1, $t_1 = 0$ and the solution corresponds to partial taxation. The threshold for $t_1 = 0$ is obtained as follows. Let $\Upsilon_{II} = w^t_{aP}/w^t_{c1}$ and $\Omega_{II} = w^t_{c2}/w^t_{c1}$, and take $(a - c_P)/c_2$ as given. The cutoff value for c_1/c_2 can be written as

$$\frac{\underline{c_1}}{c_2}(\gamma) = \Upsilon_{II}(\gamma)\frac{(a - c_P)}{c_2} + \Omega_{II}(\gamma) \quad (65)$$

Equation (65) is depicted in Figure 5 (dashed line). It is such that $\Upsilon_{II}(0) = 54/115 \simeq 0.47$, $\Omega_{II}(0) = 61/115 \simeq 0.53$, and $\frac{\underline{c_1}}{c_2}(\gamma)$ increases converging to the dotted line (that delimitates the ruled-out parameters area).

Repeating the same procedure for τ_1 and τ_2 the function equivalent to (65), i.e, that separates the positive-rate area from the zero-rate area, lies over the dotted line in Figure 5 in both cases, which is ruled out: hence, $\tau_1 > 0$ and $\tau_2 > 0$ for all relevant parameters. Finally, since t_2 increases in c_1 there is no possibility of a negative rate as c_1 grows large. This condition is also helpful to prove that $t_2 > t_1$. Also, it can be shown that $w^\tau_{c1} < w^\tau_{c2}$, and hence $\tau_1 > \tau_2$.

When parameters lie between (65) and the dotted line in Figure 5, $t_1 = 0$ and the other three rates can be obtained from (46)-(48), as follows

$$\begin{aligned}
\tau_1 &= w_{aP}^{\tau 1}(a - c_P) - w_{c1}^{\tau 1} c_1 - w_{c2}^{\tau 1} c_2 \\
t_2 &= w_{aP}^{t2}(a - c_P) + w_{c1}^{t2} c_1 - w_{c2}^{t2} c_2 \\
\tau_2 &= w_{aP}^{\tau 2}(a - c_P) - w_{c1}^{\tau 2} c_1 - w_{c2}^{\tau 2} c_2
\end{aligned}$$

where $(w_{aP}^{\tau 1}, w_{c1}^{\tau 1}, w_{c2}^{\tau 1}, w_{aP}^{t2}, w_{c1}^{t2}, w_{c2}^{t2}, w_{aP}^{\tau 2}, w_{c1}^{\tau 2}, w_{c2}^{\tau 2})$ are positive-valued functions of γ. An inspection of these equations raises the possibility that τ_1 and τ_2 may become negative if c_1 is large enough, for all γ. Again, finding the cutoff value for c_1/c_2, both for τ_1 and τ_2 reveals that function equivalent to (65), that separates the positive-rate area from the zero-rate area of the corresponding rate, lies over the dotted line in Figure 5 and is ruled out. Finally, it can be shown that $w_{c1}^{\tau 1} - w_{c2}^{\tau 1} < w_{c1}^{\tau 2} - w_{c2}^{\tau 2}$ and hence $\tau_1 > \tau_2$ within this region.

Second, asymmetries in retail cost by province and tax rates may render negative markups. After examination of all cases, the relevant mark up that may became negative is $m_{1A} = p_{1A} - p_A - t_1 - c_{1A}$. Intuitively, (i) the asymmetry is such that $c_1/c_2 > 1$ and the candidate markets that may become unprofitable are the retail markets in province 1, and (ii) province 1 diminishes t_1 (to eventually 0) and increases τ_1 (affecting good A through τ_1 more than province 2 affects product B through τ_2).

Introducing the tax rates (the four positive tax rates in the full-tax case or $t_1 = 0$ and the remaining rates in the partial-tax case) into m_{1A}, we look for the threshold c_1/c_2 such the markup is zero:

$$\begin{aligned}
\frac{\bar{c}_1}{c_2}(\gamma) &= \frac{\alpha_{II1}}{\beta_{II1}} + \frac{\psi_{II1}}{\beta_{II1}} \frac{(a - c_P)}{c_2} \qquad \text{if } \frac{c_1}{c_2} \leq \frac{\underline{c}_1}{c_2}(\gamma) \\
&= \frac{\alpha_{II2}}{\beta_{II2}} + \frac{\psi_{II2}}{\beta_{II2}} \frac{(a - c_P)}{c_2} \qquad \text{if } \frac{c_1}{c_2} > \frac{\underline{c}_1}{c_2}(\gamma)
\end{aligned} \tag{66}$$

where

$$\begin{aligned}
\psi_{II1} &= \frac{1 - 2\delta}{2 + \gamma} - (1 - \psi - 2\delta) w_{aP}^t - (\omega - \rho - \sigma + \theta) w_{aP}^\tau \\
\beta_{II1} &= \frac{1 - \delta}{2 + \gamma} - (1 - \alpha - \delta) w_{c1}^t - (\delta - \beta) w_{c2}^t - (\omega - \rho) w_{c1}^\tau + (\sigma - \theta) w_{c2}^\tau
\end{aligned}$$

$$
\alpha_{II1} = \frac{\delta}{2+\gamma} - (1-\alpha-\delta)w_{c2}^{t} - (\delta-\beta)w_{c1}^{t} + (\omega-\rho)w_{c2}^{\tau} - (\sigma-\theta)w_{c1}^{\tau}
$$

$$
\begin{aligned}
\psi_{II2} &= \frac{1-2\delta}{2+\gamma} + (\delta-\beta)w_{aP}^{t2} - (\omega-\rho)w_{aP}^{\tau 1} + (\sigma-\theta)w_{aP}^{\tau 2} \\
\beta_{II2} &= \frac{1-\delta}{2+\gamma} - (\delta-\beta)w_{c1}^{t2} - (\omega-\rho)w_{c1}^{\tau 1} + (\sigma-\theta)w_{c1}^{\tau 2} \\
\alpha_{II2} &= \frac{\delta}{2+\gamma} - (\delta-\beta)w_{c2}^{t2} + (\omega-\rho)w_{c2}^{\tau 1} - (\sigma-\omega)w_{c2}^{\tau 2}
\end{aligned}
$$

Equation (66) is depicted in Figure 5 (solid line). In the region with $t_1 = 0$ it is such that $\frac{\alpha_{II2}}{\beta_{II2}}(0) = \frac{15}{37}$, $\frac{\psi_{II2}}{\beta_{II2}}(0) = \frac{22}{37}$, and $\frac{\bar{c}_1}{c_2}(\gamma)$ increases in γ but crossing $\frac{c_1}{c_2}(\gamma)$ at $\gamma_{c2} \simeq 6.57$. At this point, it switches (continuously) to the function corresponding to the region with positive tax rates. *Q.E.D.*

B.4 Proof of Proposition 4

Assume $a_1 = a_2 = a$, $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$ and let $c_A > c_B$. The functions Γ_{1A} to Γ_{2B} defined in (28)-(31) simplify to

$$
\Gamma_{1A} = \Gamma_{1B} = \psi(a-c_R) - \frac{(1+\gamma)}{(2+\gamma)}\left(\omega + \frac{\kappa}{\psi}\right)c_A + \frac{(1+\gamma)}{(2+\gamma)}\frac{(2+3\gamma)+\gamma(1+\gamma)}{(2+3\gamma)}\theta c_B \quad (67)
$$

$$
\Gamma_{2A} = \Gamma_{2B} = \psi(a-c_R) - \frac{(1+\gamma)}{(2+\gamma)}\left(\omega + \frac{\kappa}{\psi}\right)c_B + \frac{(1+\gamma)}{(2+\gamma)}\frac{(2+3\gamma)+\gamma(1+\gamma)}{(2+3\gamma)}\theta c_A \quad (68)
$$

Replacing these values into (38)-(41), tax rates can be written as functions of $a - c_R$, c_A and c_B. Equations (42)-(45) simplify to

$$
\begin{aligned}
t_1 &= w_{aR}^{t}(a-c_R) - w_{AB}^{t}c_A + w_{BA}^{t}c_B \\
\tau_1 &= w_{aR}^{\tau}(a-c_R) - w_{AB}^{\tau}c_A - w_{BA}^{\tau}c_B \\
t_2 &= w_{aR}^{t}(a-c_R) + w_{BA}^{t}c_A - w_{AB}^{t}c_B \\
\tau_2 &= w_{aR}^{\tau}(a-c_R) - w_{BA}^{\tau}c_A - w_{AB}^{\tau}c_B
\end{aligned}
$$

where $(w_{aR}^{t}, w_{aR}^{\tau}, w_{AB}^{t}, w_{BA}^{t}, w_{AB}^{\tau}, w_{BA}^{\tau})$ are positive functions of γ. The term w_{AB}^{τ} is positive for $\gamma < \gamma_{AB}$ (where $\gamma_{AB} \simeq 10$) and negative otherwise, and always less than w_{BA}^{τ}.

First, notice that rates t_1 and τ_2 decrease in c_A and may become negative if c_A is large enough for all γ, while τ_1 may become negative if c_A is large enough, for low γ. Rate t_2 increases in c_A and hence $t_2 > t_1$. Also, $w^{\tau}_{AB}(\gamma) > w^{\tau}_{BA}(\gamma)$ and hence $\tau_1 > \tau_2$ for all γ. Moreover, τ_1 decreases (increases) in c_A for $\gamma < (>) \gamma_{AB}$.

For high values of c_A, $t_1 = 0$ and the solution corresponds to partial taxation. The threshold for $t_1 = 0$ is obtained as follows. Let $\Upsilon_{III} = w^t_{aR}/w^t_{AB}$, $\Omega_{III} = w^t_{BA}/w^t_{AB}$, and take $(a - c_R)/c_B$ as given. The cutoff value of c_A/c_B for $t_1 = 0$ can be written as s

$$\frac{\underline{c}_A}{c_B}(\gamma) = \Upsilon_{III}(\gamma)\frac{(a - c_R)}{c_B} + \Omega_{III}(\gamma) \tag{69}$$

Equation (69) is depicted in Figure 6 (dashed line). It is such that $\Upsilon_{III}(0) = 54/71 \simeq 0.76$, $\Omega_{III}(0) = 17/71 \simeq 0.24$ and $\frac{\underline{c}_A}{c_B}(\gamma)$ decreases to 1 as $\gamma \to \infty$. (The function is always below the dotted line, that delimitates the rule-out parameters area.)

Repeating the same procedure for τ_2, the function equivalent to (69), that separates the positive-rate area from the zero-rate area, lies over (69) and is ruled out: hence, $\tau_2 > 0$ for all relevant parameters. Finally, repeating the same procedure for τ_1 and $\gamma \in [0, 10]$ leads to the same conclusion, and $\tau_1 > 0$ for all relevant parameters.

Second, asymmetries in producer costs and tax rates may render negative markups. After examination of all cases, the relevant markup that may become negative is $m_{2A} = p_{2A} - p_A - t_2 - c_{2A}$. Intuitively, (i) the asymmetry is such that $c_A/c_B > 1$ and the candidate markets that may become unprofitable are the retail A markets, and (ii) the tax-mix is such that $\tau_1 > \tau_2$ (upstream tax relatively higher in the region that produces good A) and $t_2 > t_1$ (downstream tax relatively higher in region 2).[12]

Introducing the tax rates from the full-tax solution into m_{2A}, we look for the threshold c_A/c_B such that the markup is zero:

$$\frac{\bar{c}_A}{c_B}(\gamma) = \frac{\alpha_{III}}{\beta_{III}} + \frac{\psi_{III}}{\beta_{III}}\frac{(a - c_R)}{c_B} \tag{70}$$

where

$$\psi_{III} = \frac{1 - 2\delta}{2 + \gamma} - (1 - \psi - 2\delta)w^t_{aR} - (\omega - \rho - \sigma + \theta)w^{\tau}_{aR}$$

[12] The fact that $\tau_1 > \tau_2$ should not be confused with the idea of upstream taxing the product that costs more. Indeed, τ_1 decreases in c_A for low and intermediate values of γ, and slightly increases for high values of γ. At some point province 1 decreases t_1 and slightly increases τ_1 when a_2 increases; but remember that there is not much room for upstream taxation when products are homogeneous (see explanations in Proposition 1).

$$\beta_{III} = \frac{[(2+3\gamma)+\gamma(1+\gamma)]\,\omega-\gamma(1+\gamma)\theta}{(2+\gamma)(2+3\gamma)} + (1-\alpha-\delta)w^t_{BA} + (\delta-\beta)w^t_{AB} - (\omega-\rho)w^\tau_{AB} + (\sigma-\theta)w^\tau_{BA}$$

$$\alpha_{III} = \frac{\gamma(1+\gamma)\omega}{2(2+\gamma)(2+3\gamma)} + (1-\alpha-\delta)w^t_{AB} + (\delta-\beta)w^t_{BA} + (\omega-\rho)w^\tau_{BA} - (\sigma-\theta)w^\tau_{AB}$$

Equation (70) is depicted in Figure 6 (solid line). It is such that $\frac{\alpha_{III}}{\beta_{III}}(0) = \frac{4}{7}$, $\frac{\psi_{III}}{\beta_{III}}(0) = \frac{3}{7}$, $\frac{\bar{c}_A}{c_B}(\gamma)$ decreases in γ and $\frac{\bar{c}_A}{c_B}(\gamma) < \frac{\underline{c}_A}{c_B}(\gamma)$ for all γ. This last condition rules out cases with zero-tax rate and non-negative markup m_{2A}. *Q.E.D.*

B.5 Proof of proposition 5

Assume $a_1 = a_2 = a$, $c_A = c_B = c_P$, $c_{1A} = c_{2A} = c_{RA}$, $c_{1B} = c_{2B} = c_{RB}$ and let $c_{RA} \geq c_{RB}$. The functions Γ_{1A} to Γ_{2B} defined in (28)-(31) simplify to

$$\Gamma_{1A} = \Gamma_{2A} = \psi(a-c_P) - (\psi+\kappa)c_{RA} + \kappa c_{RB} \quad (71)$$

$$\Gamma_{1B} = \Gamma_{2B} = \psi(a-c_P) - (\psi+\kappa)c_{RB} + \kappa c_{RA} \quad (72)$$

Replacing these values into (38)-(41), tax rates can be written as functions of $a - c_P$, c_{RA} and c_{RB}. Equations (42)-(45) simplify to

$$t_1 = w^t_{aP}(a-c_P) - w^t_{RARB}c_{RA} - w^t_{RBRA}c_{RB}$$

$$\tau_1 = w^\tau_{aP}(a-c_P) - w^\tau_{RARB}c_{RA} + w^\tau_{RBRA}c_{RB}$$

$$t_2 = w^t_{aP}(a-c_P) - w^t_{RBRA}c_{RA} - w^t_{RARB}c_{RB}$$

$$\tau_2 = w^\tau_{aP}(a-c_P) + w^\tau_{RBRA}c_{RA} - w^\tau_{RARB}c_{RB}$$

where $(w^t_{aP}, w^\tau_{aP}, w^t_{RARB}, w^\tau_{RARB}, w^t_{RBRA}, w^\tau_{RBRA})$ are positive-valued functions of γ.

First, notice that t_1, t_2 and τ_1 may become negative as c_{RA} grows large. For high values of c_{RA}, $\tau_1 = 0$ and the solution corresponds to partial taxation. The threshold for $\tau_1 = 0$ is obtained as follows. Let $\Upsilon_{IV} = w^\tau_{aP}/w^\tau_{RBRA}$ and $\Omega_{IV} = w^\tau_{RARB}/w^\tau_{RBRA}$, and take $(a-c_P)/c_{RB}$ as given. The cutoff value for c_{RA}/c_{RB} can be written as

$$\frac{c_{RA}}{c_{RB}}(\gamma) = \Upsilon_{IV}(\gamma)\frac{(a-c_P)}{c_{RB}} + \Omega_{IV}(\gamma) \quad (73)$$

Equation 73 is depicted in Figure 7 (dashed line). It is such that $\Upsilon_{IV}(0) = 108/133 \simeq 0.81$, $\Omega_{IV}(0) = 25/133 \simeq 0.19$and $\frac{c_{RA}}{c_{RB}}(\gamma)$ decreases to 1 as $\gamma \to \infty$.
Repeating the same procedure for t_1 and t_2 the function equivalent to (73), i.e, that separates the full-tax solution from the partial-tax solution, lies over the dotted line of Figure 7 (i.e., within the area of ruled-out parameters) in both cases, and is ruled out: hence, $t_1 > 0$ and $t_2 > 0$ for all relevant parameters. Finally, since τ_2 increases in c_{RA} there is no possibility of a negative rate as c_{RA} grows large; moreover, this also proves that $\tau_2 > \tau_1$. Finally, it can be shown that $w^t_{RARB} < w^t_{RBRA}$ and hence $t_1 > t_2$.
When parameters lie between (73) and the dotted-line of Figure 7, $\tau_1 = 0$ and the other three rates can be obtained from (52)-(54), as follows

$$\begin{aligned} t_1 &= w^{t1}_{aP}(a - c_P) - w^{t1}_{RA}c_{RA} - w^{t1}_{RB}c_{RB} \\ t_2 &= w^{t2}_{cP}(a - c_P) - w^{t2}_{RA}c_{RA} - w^{t2}_{RB}c_{RB} \\ \tau_2 &= w^{\tau2}_{aP}(a - c_P) + w^{\tau2}_{RA}c_{RA} - w^{\tau2}_{RB}c_{RB} \end{aligned}$$

where $(w^{t1}_{aP}, w^{t1}_{RA}, w^{t1}_{RB}, w^{t2}_{aP}, w^{t2}_{RA}, w^{t2}_{RB}, w^{\tau2}_{aP}, w^{\tau2}_{RA}, w^{\tau2}_{RB})$ are positive-valued functions of γ. An inspection of these equations raises the possibility that t_1 and t_2 can become negative if c_{RA} is large enough, for all γ. Again, finding the cutoff value for c_{RA}/c_{RB}, both for t_1 and t_2 reveals that function equivalent to (73), that separates the positive-rate area from the zero-rate area of the corresponding rate, lies over the dotted-line of Figure 7 and is ruled out. Finally, it can be seen that $w^{t1}_{RA} - w^{t1}_{RB} < w^{t2}_{RA} - w^{t2}_{RB}$ and hence $t_1 > t_2$ within this region.

Second, asymmetries in retail costs by product and tax rates may render negative mark ups. After examination of all cases, the relevant mark up that may become negative is $m_{1A} = p_{1A} - p_A - t_1 - c_{1A}$in the case with $t_1 = 0$ (since the threshold with full-tax lies over the function (74) explained below). Intuitively, (i) the asymmetry is such that $c_{RA}/c_{RB} > 1$ and the candidate markets that may become unprofitable are the retail markets of product A, and (ii) province 1 only collects through t_1 while province 2 collects through (t_2, τ_2) in a more balanced way.
Introducing the tax rates ($\tau_1 = 0$ and the remaining positive rates) into m_{1A}, we look for the threshold c_{RA}/c_{RB} such the mark up is zero.

$$\frac{\bar{c}_{RA}}{c_{RB}}(\gamma) = \min\left\{\frac{\alpha_{IV}}{\beta_{IV}} + \frac{\psi_{IV}}{\beta_{IV}}\frac{(a-c_P)}{c_{RB}}, \frac{\kappa}{\psi+\kappa} + \frac{\psi}{\psi+\kappa}\frac{(a-c_P)}{c_{RB}}\right\} \tag{74}$$

where

$$\begin{aligned}
\psi_{IV} &= \frac{1-\delta}{2+\gamma} - (1-\alpha-\delta)w_{aP}^{t1} + (\delta-\beta)w_{aP}^{t2} + (\sigma-\theta)w_{aP}^{\tau 2} \\
\beta_{IV} &= \frac{[(2+3\gamma)+\gamma(1+\gamma)](1-2\delta)}{(2+\gamma)(2+3\gamma)} - \frac{[(2+3\gamma)+2\gamma(1+\gamma)]\theta}{(2+\gamma)(2+3\gamma)} \\
&\quad -(1-\alpha-\delta)w_{RA}^{t1} + (\delta-\beta)w_{RA}^{t2} - (\sigma-\theta)w_{RA}^{\tau 2} \\
\alpha_{IV} &= \frac{\gamma(1+\gamma)(1-2\delta)}{(2+\gamma)(2+3\gamma)} - \frac{[(2+3\gamma)+2\gamma(1+\gamma)]\theta}{(2+\gamma)(2+3\gamma)} + (1-\alpha-\delta)w_{RB}^{t1} - (\delta-\beta)w_{RB}^{t2} - (\sigma-\theta)w_{RB}^{\tau 2}
\end{aligned}$$

Equation (74) is depicted in Figure 7 (solid line). For low values of γ it is $\frac{\alpha_{IV}}{\beta_{IV}} + \frac{\psi_{IV}}{\beta_{IV}}\frac{(a-c_P)}{c_{RB}}$, such that $\frac{\alpha_{IV}}{\beta_{IV}}(0) = \frac{27}{163}$, $\frac{\psi_{IV}}{\beta_{IV}}(0) = \frac{136}{163}$, and decreases in γ but crossing the function $\frac{\kappa}{\psi+\kappa} + \frac{\psi}{\psi+\kappa}\frac{(a-c_P)}{c_{RB}}$ (which is the functional form of the dotted line) at $\gamma \simeq 8.56$. *Q.E.D.*

C Miscellaneous

C.1 Solution for cooperative taxation

This section of the Appendix proposes a solution, out of the infinite solutions that satisfy (18)-(20). It works provided that asymmetries are not too large, which is fine because the cases analyzed in Section 4 rule out large asymmetries. Depending on the combinations between (19) and (20) there are four possible cases:

- **Case 1:** $\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B} \geq 0$ and $\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B} \geq 0$. In this case we propose

$$\begin{aligned}
t_2 &= \tau_2 = \begin{array}{l} \frac{\Gamma_{1A}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{1B}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) + \\ \frac{\Gamma_{2A}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{2B}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) \end{array} \\
t_1 &= t_2 + \frac{\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B}}{4(\alpha+\beta)} \\
\tau_1 &= \tau_2 + \frac{\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B}}{4(\mu+\kappa)}
\end{aligned}$$

- **Case 2:** $\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B} < 0$ and $\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B} \geq 0$. In this case we propose

$$t_1 = \tau_2 = \begin{array}{l} \frac{\Gamma_{1A}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{1B}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) + \\ \frac{\Gamma_{2A}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{2B}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) \end{array}$$

$$
\begin{aligned}
t_2 &= t_1 + \frac{\Gamma_{2A} + \Gamma_{2B} - \Gamma_{1A} - \Gamma_{1B}}{4(\alpha + \beta)} \\
\tau_1 &= \tau_2 + \frac{\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B}}{4(\mu + \kappa)}
\end{aligned}
$$

- **Case 3:** $\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B} \geq 0$ and $\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B} < 0$. In this case we propose

$$
\begin{aligned}
t_2 &= \tau_1 = \begin{array}{l} \frac{\Gamma_{1A}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{1B}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) + \\ \frac{\Gamma_{2A}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{2B}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) \end{array} \\
t_1 &= t_2 + \frac{\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B}}{4(\alpha + \beta)} \\
\tau_2 &= \tau_1 + \frac{\Gamma_{1B} + \Gamma_{2B} - \Gamma_{1A} - \Gamma_{2A}}{4(\mu + \kappa)}
\end{aligned}
$$

- **Case 4:** $\Gamma_{1A} + \Gamma_{1B} - \Gamma_{2A} - \Gamma_{2B} > 0$ and $\Gamma_{1A} + \Gamma_{2A} - \Gamma_{1B} - \Gamma_{2B} < 0$. In this case we propose

$$
\begin{aligned}
t_1 &= \tau_1 = \begin{array}{l} \frac{\Gamma_{1A}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{1B}}{16}\left(\frac{1}{\psi} + \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) + \\ \frac{\Gamma_{2A}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} + \frac{1}{\mu+\kappa}\right) + \frac{\Gamma_{2B}}{16}\left(\frac{1}{\psi} - \frac{1}{\alpha+\beta} - \frac{1}{\mu+\kappa}\right) \end{array} \\
t_2 &= t_1 + \frac{\Gamma_{2A} + \Gamma_{2B} - \Gamma_{1A} - \Gamma_{1B}}{4(\alpha + \beta)} \\
\tau_2 &= \tau_1 + \frac{\Gamma_{1B} + \Gamma_{2B} - \Gamma_{1A} - \Gamma_{2A}}{4(\mu + \kappa)}
\end{aligned}
$$

C.2 Retail mark ups

Proofs to Propositions 2 to 3 use a condition that rules out negative retail mark ups. Specifically, given the proposed asymmetries, the non-negativity conditions must be checked on either m_{1A} or m_{2A}. Using the definition of prices from equations (3), (6) and (7), mark ups are

$$
\begin{aligned}
m_{1A} &= p_{1A} - c_{1A} - t_1 - p_A \\
&= \Psi_{1A} - \Phi_A - c_{1A} - (1 - \alpha - \delta)t_1 + (\delta - \beta)t_2 - (\omega - \rho)\tau_1 + (\sigma - \theta)\tau_2
\end{aligned}
$$

and

$$
\begin{aligned}
m_{2A} &= p_{2A} - c_{2A} - t_2 - p_A \\
&= \Psi_{2A} - \Phi_A - c_{2A} - (1 - \alpha - \delta)t_2 + (\delta - \beta)t_1 - (\omega - \rho)\tau_1 + (\sigma - \theta)\tau_2
\end{aligned}
$$

Mark up m_{1A} is used in proofs to Propositions 2 (Appendix B.2), 5 (Appendix B.5) and 3 (Appendix B.3).

- When $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$, $c_A = c_B = c_P$, and $a_2 \geq a_1$,

$$\begin{aligned}
\Phi_A &= \Phi_B = \Phi = \delta(a_1 + a_2) - 2\delta c_R + (1 - 2\delta)c_P \\
\Psi_{1A} &= \frac{a_1}{(2+\gamma)} + \frac{(1+\gamma)}{(2+\gamma)}(\Phi + c_R) \\
\Psi_{1A} - \Phi_A - c_{1A} &= \frac{(1-\delta)}{(2+\gamma)}a_1 - \frac{\delta}{(2+\gamma)}a_2 - \frac{(1-2\delta)}{(2+\gamma)}(c_P + c_R)
\end{aligned} \tag{75}$$

- When $a_1 = a_2 = a$, $c_A = c_B = c_P$, $c_{1A} = c_{2A} = c_{RA}$, $c_{1B} = c_{2B} = c_{RB}$ and $c_{RA} \geq c_{RB}$,

$$\Psi_{1A} - \Phi_A - c_{1A} = \omega_{aP}(a - c_P) - \omega_{RA}c_{RA} + \omega_{RB}c_{RB}$$

where

$$\begin{aligned}
\omega_{aP} &= \frac{1-2\delta}{2+\gamma} > 0 \\
\omega_{RA} &= \frac{[(2+3\gamma) + \gamma(1+\gamma)](1-2\delta)}{(2+\gamma)(2+3\gamma)} - \frac{[(2+3\gamma) + 2\gamma(1+\gamma)]\theta}{(2+\gamma)(2+3\gamma)} > 0 \\
\omega_{RB} &= \frac{\gamma(1+\gamma)(1-2\delta)}{(2+\gamma)(2+3\gamma)} - \frac{[(2+3\gamma) + 2\gamma(1+\gamma)]\theta}{(2+\gamma)(2+3\gamma)} > 0
\end{aligned}$$

- When $a_1 = a_2 = a$, $c_A = c_B = c_P$, $c_{1A} = c_{1B} = c_1$, $c_{2A} = c_{2B} = c_2$ and $c_1 \geq c_2$,

$$\Psi_{1A} - \Phi_A - c_{1A} = \frac{(1-2\delta)}{(2+\gamma)}(a - c_P) - \frac{(1-\delta)}{(2+\gamma)}c_1 + \frac{\delta}{(2+\gamma)}c_2$$

Mark up m_{2A} is used in proof to Proposition 4 (Appendix B.4).

- When $a_1 = a_2 = a$, $c_{1A} = c_{1B} = c_{2A} = c_{2B} = c_R$ and let $c_A > c_B$.

$$\begin{aligned}
\Phi_A &= 2\delta\,(a - c_R) + \omega c_A + \theta c_B \\
\Phi_B &= 2\delta\,(a - c_R) + \theta c_A + \omega c_B \\
\Psi_{2A} &= \frac{a + (1+\gamma)c_R}{(2+\gamma)} + \frac{(1+\gamma)\,[2(1+\gamma)\Phi_A + \gamma\Phi_B]}{(2+\gamma)(2+3\gamma)} \\
\Psi_{2A} - \Phi_A - c_R &= \omega_{aR}(a - c_R) - \omega_A c_A + \omega_B c_B
\end{aligned}$$

where

$$\begin{aligned}
\omega_{aR} &= \frac{1-2\delta}{2+\gamma} > 0 \\
\omega_A &= \frac{[(2+3\gamma) + \gamma(1+\gamma)]\,\omega - \gamma(1+\gamma)\theta}{(2+\gamma)(2+3\gamma)} > 0 \\
\omega_B &= \frac{\gamma(1+\gamma)\omega}{2(2+\gamma)(2+3\gamma)} > 0
\end{aligned}$$

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