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Multi-stage Taxation by Subnational gGovernments: Welfare Effects

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Abstract

This paper analyzes multi-stage taxation by provinces in a federal country, using a two-good, two-province, two-stage successive differentiated-product symmetric oligopoly model, where each producer is located in a province and sells its product through exclusive retailers located in both provinces. Retailers compete for consumers a la Bertrand with differentiated products. The producer-retailer setup allows provincial governments to raise taxes on both upstream and downstream links of the value chain.

We solve a simultaneous and non-cooperative tax competition problem, where (symmetric) provinces choose tax rates to maximize welfare subject to a revenue constraint. We find that provinces set tax rates to either raise revenue at only one segment of the value chain or use a combination of upstream and downstream taxation. This choice is determined by the revenue requirement, the size of the market and the degree of downstream competition. We characterize and discuss each possible case.

Comparing the results of this model with the Leviathan case (analyzed in a previous paper by the authors) where governments behave as revenue maximizers, we find that there is a threshold on revenue requirement such that welfarist governments tend to behave qualitatively similar to Leviathan governments when revenue need exceed the threshold: they both choose a combination of taxes if products have some degree of heterogeneity, whereas they rely on downstream taxation when products are homogeneous. This way we provide a rationale for raising taxes on successive taxation even when governments internalize the effect of successive taxation on welfare.

Keywords: local indirect taxation, multistage taxes, tax competition, welfare taxation.

JEL Codes: H71, H21, H22.

Multi-stage taxation by subnational governments: Welfare effects

1 Introduction

General taxes on goods and services are an important source of fiscal revenue. They are top–of–the–list within indirect taxes, and are typically classified into value-added –or consumption– taxes, sales taxes, multi-stage cumulative taxes, excises, taxes on trade, etc. (following OECD's classification). Other important sources of revenue are taxes on income (direct taxation on corporate or personal income or earnings) and property (direct taxation on wealth).

When the power to raise revenues is vested on a centralized level, fiscal authorities prefer VAT or sales taxes over multi-stage taxes. There is a reason for that: multi-stage taxation creates inefficiency along the value-chain (the so-called "cascade" effect).[1] Governments naturally internalize the double-margin effect from multi-stage taxation and prefer consumption or sales taxes over turnover taxes. In fact, countries that joined the European Community used to raise some form of turnover taxes, and replaced them for VAT during the mid-80s (Tait, 1988).

But multi-stage taxes arise as a preferred instrument at provincial levels, and although they are not widespread their popularity has been increasing in later years. For example, in Argentina provincial governments collect turnover taxes (*impuesto sobre los ingresos brutos*), which represent more than 70 percent of resources collected at provincial level.[2] Moreover, different versions of successive taxes are entering the scene again in developed federal countries. For example, in the US, the state of Washington collects a Business and Occupations tax, Ohio replaced a net income tax for a Commercial Activity Tax in

[1] The cascade effect has long been studied by the public finance literature. See Friedlaender (1967).

[2] We refer the reader to many papers of this tax in Argentina. See, for example, Núñez Miñana (1994), Libonatti (1998), Piffano (2005), Porto, Garriga and Rosales (2014) and the references therein.

2014, and Nevada passed a version of Gross Receipt Tax in 2015. The current debate in Connecticut and other states suggest that this tax will gain relevance in the future (Ebel *et al.*, 2016).

A previous paper (Cont and Fernández Felices, 2016) analyzes whether successive taxation raised at provincial level can be optimal considering non-cooperative revenue maximizing governments (Leviathan) in a federal country. In the symmetric case (equal market sizes, retail costs and producer costs for both products) with no competition at the downstream level, we find a double taxation mark-up result -which mimics the standard producer-retailer relationship obtained in the industrial organization literature- with upstream rates doubling downstream rates in equilibrium. As downstream competition gets stronger, provincial governments gradually switch from upstream to downstream taxation, eventually abandoning upstream taxation completely under product homogeneity. This result provides a strong ground for using successive taxation when provincial governments need to raise resources. The paper also obtains an over-taxation result (which is standard in the non-cooperative taxation literature).

This companion paper explores the robustness of these results when provincial governments maximize welfare of the agents residing in the province, subject to a revenue requirement. In that sense, governments trade-off tax collection effects (obtained in a Leviathan context) with the inefficiencies generated by distortionary taxes. We use the same setup from Cont and Fernández Felices (2016): a two-good, two-stage (successive oligopoly), two-province model, where each producer is located in a province and sells its product through exclusive retailers located in both provinces. Local governments raise taxes on wholesale/upstream and retail/downstream transactions that take place in their province.

We solve a welfare taxation problem, i.e., assuming symmetric provinces that seek to maximize local welfare non-cooperatively, subject to a given (equal for both provinces) revenue target. Then we study 1) the role of upstream and downstream tax rates on revenue (i.e., whether the extent to which they are substitutes); 2) the optimal upstream-downstream mix of tax rates under welfare taxation and the determinants of such a mix; and 3) in particular, the effect of product differentiation and revenue target on the mix.

The rest of the paper is organized as follows. Section 2 presents the market setup, which is a symmetric version of the model introduced by Cont and Fernández Felices (2016), and characterizes the equilibrium prices and quantities at retail and producer levels. Section 3 solves the non-cooperative welfare maximization problem, characterizes the solution, and discusses in detail the properties of optimal tax rates under different degrees of product differentiation and revenue requirements. Section 4 compares the solution of this model with the one obtained with Leviathan (revenue maximizers) governments. Section 5 concludes.

2 Setup: model, equilibrium, taxes

2.1 The Model

Assume two producers of products A and B. Producer A is located in region / province 1, whereas producer B is located in region / province 2. The length of time is such that location is given, i.e., producers cannot move across provinces. In order to reach consumers, producers sell their products to retailers in an *upstream* market. Retailers sell products to consumers in a *downstream* market. The structure is as follows.[3]

Producer A (located in region 1) sells its product to retailer $1A$ in region 1 (x_{1A}) and retailer $2A$ in region 2 (x_{2A}) at price p_A. Total quantity of product A is $x_A = x_{1A} + x_{2A}$. Likewise, producer B (located in region 2) sells products to retailer $1B$ in region 1 and retailer $2B$ in region 2 at price p_B. We assume that producers do not discriminate prices between regions. Total quantity of product B is $x_B = x_{1B} + x_{2B}$.

[3] This paper considers a symmetric taxation problem. See Cont and Fernández Felices (2016) for a full asymmetric model.

Retailers in market $i = 1, 2$ sell to consumers with demand

$$x_{ij} = a - p_{ij} - \gamma\left(p_{ij} - p_{ik}\right) \tag{1}$$

where subscript i corresponds to provinces (1 or 2) and subscripts j, k correspond to products (either A, B or B, A). We assume a simple demand (1) (as that used by in Davidson and Deneckeree, 1985, to study merger among firms competing with differentiated products) to characterize the equilibrium based on market size (a) and the degree of downstream market-power linked to product differentiation (γ). Consumer surplus CS_{ij} associated to demand (1) is $CS_{ij} = x_{ij}^2/2(1+\gamma)$. Retailers' costs are $CT_j(x_{ij}) = (p_j + c_R)\, x_{ij}$ and producers have marginal cost c_P.

There are regional / provincial governments that collect sales taxes. They can set rates on upstream sales (τ_i) and / or rates on downstream sales (t_i) within their jurisdiction. Tax revenues in provinces 1 and 2 are

$$\begin{aligned} R_1 &= \tau_1 x_A + t_1\left(x_{1A} + x_{1B}\right) = (\tau_1 + t_1)x_{1A} + t_1 x_{1B} + \tau_1 x_{2A} \\ R_2 &= \tau_2 x_B + t_2\left(x_{2A} + x_{2B}\right) = (\tau_2 + t_2)x_{2B} + t_2 x_{2A} + \tau_2 x_{1B} \end{aligned}$$

Revenues are collected on upstream and downstream sales of units produced and sold within the same province, upstream sales of units produced in the province but sold in another province, and downstream sales of units produced in other provinces and sold in the government's province. Sections 2.2 and 2.3 characterize the market equilibrium and payoffs. Section 3 analyzes the taxation problem with local governments collecting taxes following a welfare objective.

2.2 Retailers equilibrium

Let the subindex j (k) stand for product j (k, respectively). Retailers' profit is

$$\pi_{ij} = \left(p_{ij} - c_R - t_i - p_j\right)\left[a - p_{ij} - \gamma(p_{ij} - p_{ik})\right]$$

Profit maximization and equilibrium at the downstream level leads to the following prices and quantities

$$\begin{aligned} p_{ij} &= \frac{a}{(2+\gamma)} + \frac{(1+\gamma)(c_R+t_i)}{(2+\gamma)} + \frac{(1+\gamma)\left[2(1+\gamma)p_j + \gamma p_k\right]}{(2+\gamma)(2+3\gamma)} \\ x_{ij} &= \frac{(1+\gamma)(a-c_R-t_i-p_j)}{(2+\gamma)} + \frac{\gamma(1+\gamma)^2\left(p_k-p_j\right)}{(2+\gamma)(2+3\gamma)} \end{aligned}$$

2.3 Producers equilibrium

Producer A's profit is $\pi_A^P = (p_A - \tau_1 - c_P)[x_{1A} + x_{2A}]$, i.e.,

$$\pi_A^P = (p_A - \tau_1 - c_P)\left[\frac{(1+\gamma)(2a-t_1-t_2-2c_R-2p_A)}{(2+\gamma)} + \frac{2\gamma(1+\gamma)^2\left(p_B-p_A\right)}{(2+\gamma)(2+3\gamma)}\right]$$

Producer B's profit is $\pi_B^P = (p_B - \tau_2 - c_P)[x_{1B} + x_{2B}]$, i.e.,

$$\pi_B^P = (p_B - \tau_2 - c_P)\left[\frac{(1+\gamma)(2a-t_1-t_2-2c_R-2p_B)}{(2+\gamma)} + \frac{2\gamma(1+\gamma)^2\left(p_A-p_B\right)}{(2+\gamma)(2+3\gamma)}\right]$$

Profit maximization and equilibrium at the upstream level leads to the following solution:

$$p_A = \Phi - \delta(t_1+t_2) + \omega\tau_1 + \theta\tau_2 \qquad p_B = \Phi - \delta(t_1+t_2) + \theta\tau_1 + \omega\tau_2 \tag{2}$$

$$p_{1A} = \Psi + \alpha t_1 - \beta t_2 + \rho\tau_1 + \sigma\tau_2 \qquad p_{1B} = \Psi + \alpha t_1 - \beta t_2 + \sigma\tau_1 + \rho\tau_2 \tag{3}$$

$$p_{2A} = \Psi - \beta t_1 + \alpha t_2 + \rho\tau_1 + \sigma\tau_2 \qquad p_{2B} = \Psi - \beta t_1 + \alpha t_2 + \sigma\tau_1 + \rho\tau_2 \tag{4}$$

$$x_{1A} = \Gamma - \alpha t_1 + \beta t_2 - \mu\tau_1 + \kappa\tau_2 \qquad x_{1B} = \Gamma - \alpha t_1 + \beta t_2 + \kappa\tau_1 - \mu\tau_2 \tag{5}$$

$$x_{2A} = \Gamma + \beta t_1 - \alpha t_2 - \mu\tau_1 + \kappa\tau_2 \qquad x_{2B} = \Gamma + \beta t_1 - \alpha t_2 + \kappa\tau_1 - \mu\tau_2 \tag{6}$$

Constants (Φ, Ψ, Γ) characterize the producer price, retail price and quantities in the no-tax equilibrium, and are defined in Appendix 6.1, equations (21)–(23). Parameters $(\alpha, \beta, \mu, \kappa, \rho, \sigma, \delta, \theta, \omega)$ summarize tax incidence of corresponding rates, and are defined in Appendix 6.1, equations (15)–(20). Cont and Fernández Felices (2016) explores tax incidence results for this model, which depend on the product-differentiation parameter γ.

3 Welfare problem

We analyze the case of local (provincial) benevolent welfare functions

$$\begin{aligned}\max W_1 &= CS_{1A} + CS_{1B} + \pi_A + \pi_{1A} + \pi_{1B} + R_1 \quad \text{s.t. } R_1 = R \\ \max W_2 &= CS_{2A} + CS_{2B} + \pi_B + \pi_{2A} + \pi_{2B} + R_2 \quad \text{s.t. } R_2 = R\end{aligned}$$

Notice that we set the same target on tax revenues –R–, for tractability. We want to explore how provinces choose upstream and downstream tax rates, looking for tax revenue in their own region or the other provincial region. A comparison with the Leviathan solution is left to Section 4.

Replacing (2)-(6) into profits, consumer surplus and tax revenue, welfare in province 1 is

$$\begin{aligned}W_1 = &\frac{1}{2(1+\gamma)}(\psi\Lambda - \alpha t_1 + \beta t_2 - \mu\tau_1 + \kappa\tau_2)^2 + \frac{1}{2(1+\gamma)}(\psi\Lambda - \alpha t_1 + \beta t_2 + \kappa\tau_1 - \mu\tau_2)^2 \\ &+ (2\delta\Lambda - \delta(t_1+t_2) + \theta\tau_2 - (1-\omega)\tau_1)(2\psi\Lambda - \psi t_1 - \psi t_2 - 2\mu\tau_1 + 2\kappa\tau_2) \\ &+ \left(\frac{\psi}{1+\gamma}\Lambda - (1-\alpha-\delta)t_1 + (\delta-\beta)t_2 - (\omega-\rho)\tau_1 + (\sigma-\theta)\tau_2\right)(\psi\Lambda - \alpha t_1 + \beta t_2 - \mu\tau_1 + \kappa\tau_2) \\ &+ \left(\frac{\psi}{1+\gamma}\Lambda - (1-\alpha-\delta)t_1 + (\delta-\beta)t_2 - (\omega-\rho)\tau_2 + (\sigma-\theta)\tau_1\right)(\psi\Lambda - \alpha t_1 + \beta t_2 + \kappa\tau_1 - \mu\tau_2) \\ &+\tau_1(2\psi\Lambda - \psi t_1 - \psi t_2 - 2\mu\tau_1 + 2\kappa\tau_2) + t_1(2\psi\Lambda - 2\alpha t_1 + 2\beta t_2 - \psi\tau_1 - \psi\tau_2)\end{aligned}$$

where tax revenue is

$$R_1 = \tau_1(2\psi\Lambda - \psi t_1 - \psi t_2 - 2\mu\tau_1 + 2\kappa\tau_2) + t_1(2\psi\Lambda - 2\alpha t_1 + 2\beta t_2 - \psi\tau_1 - \psi\tau_2)$$

Welfare in province 2 is

$$\begin{aligned}W_2 = &\frac{1}{2(1+\gamma)}(\psi\Lambda + \beta t_1 - \alpha t_2 - \mu\tau_1 + \kappa\tau_2)^2 + \frac{1}{2(1+\gamma)}(\psi\Lambda + \beta t_1 - \alpha t_2 + \kappa\tau_1 - \mu\tau_2)^2 \\ &+ (2\delta\Lambda - \delta(t_1+t_2) + \theta\tau_1 - (1-\omega)\tau_2)(2\psi\Lambda - \psi t_1 - \psi t_2 + 2\kappa\tau_1 - 2\mu\tau_2) \\ &+ \left(\frac{\psi}{1+\gamma}\Lambda - (1-\alpha-\delta)t_2 + (\delta-\beta)t_1 - (\omega-\rho)\tau_1 + (\sigma-\theta)\tau_2\right)(\psi\Lambda + \beta t_1 - \alpha t_2 - \mu\tau_1 + \kappa\tau_2) \\ &+ \left(\frac{\psi}{1+\gamma}\Lambda - (1-\alpha-\delta)t_2 + (\delta-\beta)t_1 - (\omega-\rho)\tau_2 + (\sigma-\theta)\tau_1\right)(\psi\Lambda + \beta t_1 - \alpha t_2 + \kappa\tau_1 - \mu\tau_2) \\ &+\tau_2(2\psi\Lambda - \psi t_1 - \psi t_2 + 2\kappa\tau_1 - 2\mu\tau_2) + t_2(2\psi\Lambda + 2\beta t_1 - 2\alpha t_2 - \psi\tau_1 - \psi\tau_2)\end{aligned}$$

where tax revenue is

$$R_2 = \tau_2(2\psi\Lambda - \psi t_1 - \psi t_2 + 2\kappa\tau_1 - 2\mu\tau_2) + t_2(2\psi\Lambda + 2\beta t_1 - 2\alpha t_2 - \psi\tau_1 - \psi\tau_2)$$

Let h_1 (h_2) be the Lagrange multiplier for province 1 (2) revenue constraint. The first order conditions to welfare maximization are

$$\begin{aligned}
\frac{\partial L_1}{\partial t_1} &= \frac{\partial CS_{1A}}{\partial t_1} + \frac{\partial CS_{1B}}{\partial t_1} + \frac{\partial \pi_{1A}}{\partial t_1} + \frac{\partial \pi_{1B}}{\partial t_1} + \frac{\partial \pi_A}{\partial t_1} + (1+h_1)\frac{\partial R_1}{\partial t_1} \leq 0; \text{ if } <, \ t_1 = 0 \\
\frac{\partial L_1}{\partial \tau_1} &= \frac{\partial CS_{1A}}{\partial \tau_1} + \frac{\partial CS_{1B}}{\partial \tau_1} + \frac{\partial \pi_{1A}}{\partial \tau_1} + \frac{\partial \pi_{1B}}{\partial \tau_1} + \frac{\partial \pi_A}{\partial \tau_1} + (1+h_1)\frac{\partial R_1}{\partial \tau_1} \leq 0; \text{ if } <, \ \tau_1 = 0 \\
\frac{\partial L_1}{\partial h_1} &= R_1 - R = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L_2}{\partial t_2} &= \frac{\partial CS_{2A}}{\partial t_2} + \frac{\partial CS_{2B}}{\partial t_2} + \frac{\partial \pi_{2A}}{\partial t_2} + \frac{\partial \pi_{2B}}{\partial t_2} + \frac{\partial \pi_B}{\partial t_2} + (1+h_2)\frac{\partial R_2}{\partial t_2} \leq 0; \text{ if } <, \ t_2 = 0 \\
\frac{\partial L_2}{\partial \tau_2} &= \frac{\partial CS_{2A}}{\partial \tau_2} + \frac{\partial CS_{2B}}{\partial \tau_2} + \frac{\partial \pi_{2A}}{\partial \tau_2} + \frac{\partial \pi_{2B}}{\partial \tau_2} + \frac{\partial \pi_B}{\partial \tau_2} + (1+h_2)\frac{\partial R_2}{\partial \tau_2} \leq 0; \text{ if } <, \ \tau_2 = 0 \\
\frac{\partial L_2}{\partial h_2} &= R_2 - R = 0
\end{aligned}$$

Combining the first-order conditions of province 1 with respect to t_1 and τ_1, and assuming interior solution for both instruments, it is possible to obtain an easy interpretation for the optimal balance of instruments for that province:

$$\frac{\frac{dCS_{1A}}{dt_1} + \frac{dCS_{1B}}{dt_1} + \frac{d\pi_{1A}}{dt_1} + \frac{d\pi_{1B}}{dt_1} + \frac{d\pi_A}{dt_1} + \frac{dR_1}{dt_1}}{\frac{dR_1}{dt_1}} = \frac{\frac{dCS_{1A}}{d\tau_1} + \frac{dCS_{1B}}{d\tau_1} + \frac{d\pi_{1A}}{d\tau_1} + \frac{d\pi_{1B}}{d\tau_1} + \frac{d\pi_A}{d\tau_1} + \frac{dR_1}{d\tau_1}}{\frac{dR_1}{d\tau_1}} \tag{7}$$

The numerator on each side of this equality represents the change in welfare of province 1 due to a marginal change in one of the tax rates. The expression indicates that province 1 should set tax rates so that the change in welfare per dollar of tax revenue is the same. This is a familiar result from the optimal taxation literature. A similar condition can be obtain for province 2.

Given the symmetry of the problem and using the parameters of the model, the first order conditions reduce to

$$t : -\left\{\frac{4\alpha\psi}{1+\gamma} + 4\delta\psi + 2\,(1-\alpha-\delta)\,\psi\right\}(\Lambda - t - \tau) + (1+h)\,(2\psi\Lambda - 2(\alpha+\psi)t - 3\psi\tau) \leq 0 \tag{8}$$

$$\tau : -\left\{\frac{3\psi^2}{1+\gamma} + 4\delta\mu + 2(1-\omega)\psi\right\}(\Lambda - t - \tau) + (1+h)\,(2\psi\Lambda - 2(\mu+\psi)\tau - 3\psi t) \leq 0 \tag{9}$$

$$h : 2\psi(t+\tau)\,(\Lambda - t - \tau) - R = 0 \tag{10}$$

with equality of (8) or (9) if either t or τ are positive. Three observations are in order. First, expression (10) reveals that τ and t are perfect substitutes as instruments on the revenue side. That is, from (10) we have[4]

$$t + \tau = \frac{\Lambda - \sqrt{\Lambda^2 - \frac{2R}{\psi}}}{2} \tag{11}$$

Second, equation (11) reveals that there is a bound on the maximum revenue obtainable by provinces, given market size (Λ) and the market conditions (ψ). Revenue requirement R cannot exceed $\psi\Lambda^2/2$. Figure 1 shows the combination of parameters γ and R for which a solution to welfare taxation exists. The solid black line represents this upper bound on tax revenues given the differentiation parameter γ. The upper bound increases with the parameter γ as market power decreases with the degree of homogeneity of products, and hence the governments have more room to collect taxes.

Third, given that equation (11) defines $(t+\tau)$ as a function of R, the optimal combination of tax rates is determined by (8) and (9). It is useful to find the simplified version of condition (7) to characterize the solution to the taxation problem:

$$t = F_\Lambda \Lambda - F_\tau \tau \tag{12}$$

[4] Equation (10) is quadratic on $t+\tau$, and hence has two solutions: the one shown in (11) and another in which the root is added to Λ. Starting from the solution located at the good side of the Laffer curve neither government finds profitable to increase tax rates. On the other hand, starting from the other possible solution (wrong side of the Laffer curve), governments find a profitable deviation by decreasing tax rates (which increase both revenue and welfare).

where $F_\Lambda = F_\Lambda(\gamma)$ and $F_\tau = F_\tau(\gamma)$. That is, in a (possibly) interior solution equation (12) shows a linear relationship between tax rates. A complete definition of F_Λ and F_τ is provided in equations (24) and (25) in Appendix 6.2.

Using equations (11) and (12) we can state the main proposition of this paper

Proposition 1 *The solution to the welfare problem is a combination of tax rates t and τ such that (11) and (12) are satisfied. There are three possible outcomes: $(0, \tau)$, $(t, 0)$ or (t, τ).*

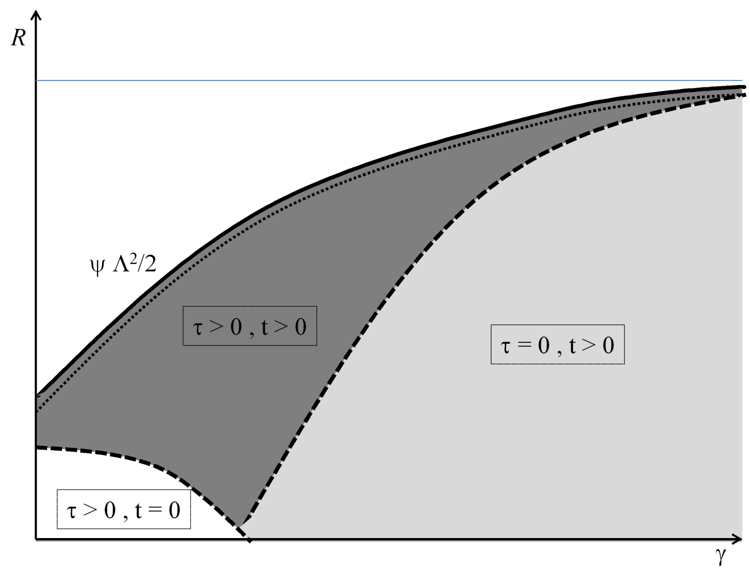
Proof: The proof is straightforward. From (11) and (12) it is easy to deduct that either the solution is interior (both rates that constitute the solution to both equations are positive) or corner (in which case either t or τ is zero). The characterization of the three possibilities is discussed below. *Q.E.D.*

According to this result, symmetric provinces setting tax rates simultaneously and non-cooperatively in order to maximize welfare subject to a given revenue constraint will find optimal to either raise revenue at only one link of the value chain (upstream or downstream) or use a combination of upstream and downstream taxation. This choice will be determined by the revenue requirement (R), the size of the market (Λ), and the degree of downstream competition (γ). Given the generality of the result, we proceed to characterize the solution for different levels of revenue requirement.

Corollary 1 *If revenue requirement is low ($R \to 0$) governments raise upstream taxes for low values of γ and downstream taxes for high values of γ. Provinces do not mix tax rates.*

This corollary states that for very low revenue requirements mixing tax rates is not part of an equilibrium. The explanation is as follows. Suppose that provincial governments need to raise taxes to get a (very low level of) revenue R. Then they have to decide on

Figure 1: **Solution to welfare taxation (as a function of γ and R).**



whether set a positive upstream or downstream rate (or combination of both). Table 1 summarizes the welfare effects of a marginal change in taxes at $t = \tau = 0$:

This table shows that the effect of t (τ) is stronger on agents who are "closer" to the tax, and these are the consumers and the retailers (the producers). When products are differentiated (low γ) a downstream tax rate has a higher negative impact on welfare than an upstream rate because final prices are higher,[5] favoring this way the use of upstream rates. But when product are more homogeneous (high γ) taxing the upstream segment unlevels the playing field, affecting negatively the good produced in the province that sets the upstream rate. This effect offsets the joint effect of downstream rate on consumers and

[5] Suppose province 1 sets a \$1 tax rate. The effect on final prices in this province is equal to 2α (6/8 if $\gamma = 0$) if the tax is set on downstream transactions, while it is equal to $\rho + \sigma$ (1/4 if $\gamma = 0$) if the tax is set on upstream transactions. Of course, there is an additional effect not captured by final prices in province 1, corresponding to the quantities of product A in province 2 (included in π_A). An increase in t_1 (τ_1) has an indirect positive (negative) effect on x_{2A}, but this effect is dominated by the direct effect on final prices when γ is low.

Table 1: Welfare incidence of downstream and upstream taxes at $t = \tau = 0$

	Effect of t	> or <	Effect of τ
Consumer surplus	$-\frac{2\alpha\psi}{(1+\gamma)}$	$<<$	$-\frac{\psi^2}{(1+\gamma)}$
Producer profit	$-4\delta\psi$	$>>$	$-2\left[(1-\omega)\psi + 2\mu\delta\right]$
Retailers profit	$-2\psi\left[(1-\alpha-\delta) + \frac{\alpha}{1+\gamma}\right]$	$<<$	$-2\frac{\psi^2}{(1+\gamma)}$
Revenue	2ψ	$=$	2ψ

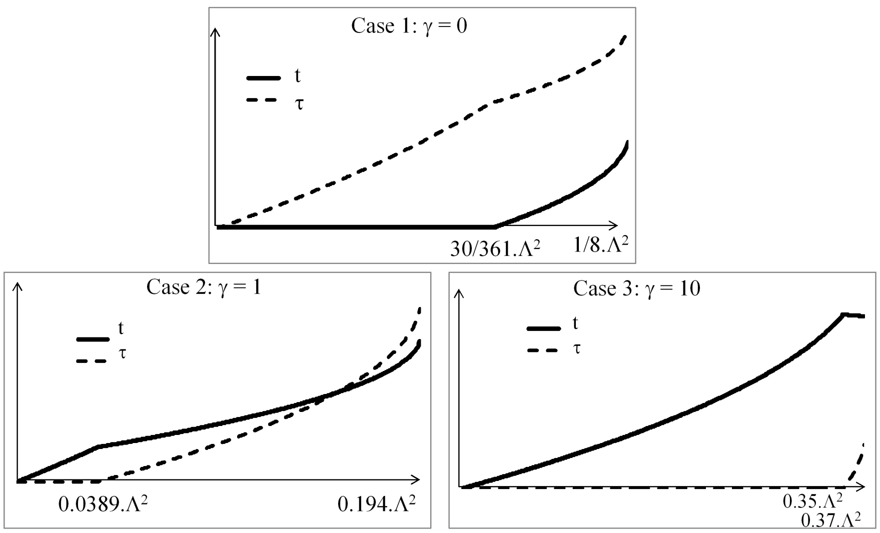
retailers. Given the structure and parameters of the model, if $\gamma < (>) 3/4$ governments start raising taxes by setting upstream (downstream) rates.

Corollary 2 *If products have some degree of heterogeneity ($\gamma < \infty$) and revenue requirement is above a certain threshold ($\underline{R}(\gamma)$), governments combine both upstream and downstream taxation. If products are homogeneous ($\gamma \to \infty$), governments use only downstream taxation.*

The dashed line in Figure 1 represents the threshold $\underline{R}(\gamma)$, which decreases with γ up to a certain value (near 3/4) and then increases in γ. Suppose local governments face market conditions with low levels of γ (say, below 3/4). Then, they will find optimal to use only upstream taxation up to the corresponding $\underline{R}(\gamma)$ (white area in Figure 1). While in this area, as R is below the threshold, an increase in R forces local authorities to increase upstream taxes to meet the constraint, which generates a negative effect on welfare. At some level of R, the negative effect on welfare equals the negative effect of downstream taxes at $t = 0$. For higher values of R governments will find beneficial to start combining both tax instruments. This is represented by the dark grey area in Figure 1.

In the specific case of $\gamma = 0$, the value $\underline{R}(0)$ that divides the area between upstream taxation and a combination of taxes is $\frac{30}{361}\Lambda^2$ (given a maximum revenue of $\Lambda^2/8$). At the highest possible requirement ($R = 1/8.\Lambda^2$), the ratio τ/t is $24/11 \sim 2.2$. See Case 1 in Figure 2 (the three cases mentioned here are characterized in Appendix C).

Figure 2: **Three cases of welfare taxation: $\gamma = 0$, $\gamma = 1$ and $\gamma = 10$.**



A similar reasoning applies for market conditions such that $\gamma > 3/4$. In such cases, when revenue requirements are below $\underline{R}(\gamma)$, it is optimal to collect taxes only at the retail stage. Increases in R in this region (light gray) implies a higher t, which reduces welfare increasingly, up to the point that governments switch to mixing rates (dark gray area). See Cases 2 and 3 in Figure 2. For example, if $\gamma = 1$, the value $\underline{R}(1)$ that divides the area between downstream taxation and a tax mix is $0.0389.\Lambda^2$ (given a maximum revenue of $0.194.\Lambda^2$). At the highest revenue requirement, the ratio τ/t is about $214/171 \sim 1.25$. When products are less heterogeneous ($\gamma = 10$), the value $\underline{R}(10)$ that divides the area between downstream taxation and a tax mix is $0.35.\Lambda^2$ (given a maximum revenue of $0.37.\Lambda^2$). Notice that as γ grows large the room for upstream rates diminishes significantly. Indeed, at the highest revenue requirement, the ratio τ/t is about $12/38 \sim 0.26$. Finally, as $\gamma \to \infty$ (homogeneous products) the only instrument to raise taxes is at the retail level.

4 Comparison with the Leviathan problem

In Cont and Fernández Felices (2016) we analyzed the problem of two provincial governments raising taxes pursuing a goal of revenue maximization. We argued that in a competitive environment between provinces, Leviathan governments use a mix of taxes that depends on the degree of market competition. The tax mix is shown in Proposition 2 for the symmetric case:

Proposition 2 *Symmetric Leviathan solution. Equilibrium rates are $t_1^S = t_2^S = t^S$ satisfying (13) and $\tau_1^S = \tau_2^S = \tau^S$ satisfying (14).*

$$t^S = \frac{2\psi(2\mu - \psi)}{4(2\alpha - \beta)(2\mu - \kappa) - 9\psi^2}\Lambda \tag{13}$$

$$\tau^S = \frac{2\psi(2\alpha - \psi)}{4(2\alpha - \beta)(2\mu - \kappa) - 9\psi^2}\Lambda \tag{14}$$

Proof: See Cont and Fernández Felices (2016).

The main contrasts between the welfare and Leviathan solutions are as follows. First, in the Leviathan symmetric solution, governments set a combination of taxes. This combination tilts towards upstream rates for low values of γ and towards downstream for high levels of γ. Second, Leviathan governments never rely 100% on upstream taxes, while Welfarist governments only rest on upstream taxes when both revenue requirements and the degree of competition are low. Third, under different objectives (Welfare or Leviathan) governments raise 100% retail taxes when the degree of competition is high (in the Welfare case, regardless of the revenue requirement). Fourth, Leviathan governments tax in excess and achieve a revenue of

$$\frac{8\psi^2(\mu+\alpha-\psi)(4\alpha\mu-\psi^2)}{[4(2\alpha-\beta)(2\mu-\kappa)-9\psi^2]^2}\Lambda^2$$

which is slightly less than $\psi\Lambda^2/2$ (this is represented in Figure 1 by the dotted line slightly below the black solid line). This result is standard in the literature of non-cooperative tax setting. Therefore, the maximum tax revenue cannot be achieved in the Leviathan competitive solution. Lastly, when the revenue requirement is high, Welfarist governments tend to behave qualitatively similar to Leviathan governments. For example, the tax ratio τ/t is approximately 2.2 for $\gamma = 0$ (2 in the Leviathan case), 1.25 for $\gamma = 1$ (1.27 in the Leviathan case) and 0.26 for $\gamma = 10$ (0.35 in the Leviathan case).

5 Conclusions and future work

This paper analyzes multi-stage taxation by provinces in a federal country. In order to do so, we set up a two-good, two-province, two-stage successive differentiated-product oligopoly model, where each producer is located in a province and sells its product through exclusive retailers located in both provinces. Retailers compete for consumers a la Bertrand with differentiated products. The producer-retailer setup allows provincial governments to raise taxes on both wholesale / upstream and retail / downstream transactions. With this model, we study Welfare taxation and compare results with the Leviathan case analyzed in Cont and Fernández Felices (2016). We show that symmetric provinces choosing tax rates simultaneously and non-cooperatively in order to maximize welfare subject to a given revenue constraint find optimal to either collect taxes at only one segment of the value chain (upstream or downstream) or use a combination of upstream and downstream taxation. This choice will be determined by the revenue requirement, the size of the market and the degree of downstream competition. For a given revenue requirement, upstream and downstream taxes are perfect substitutes on the revenue side, so the optimal mix ultimately depends on the relative effects of each instrument on consumer surplus and on profits obtained by producers and retailers.

When provinces need to raise low levels of revenue, they choose to collect taxes only at the producer level when the degree of downstream competition is sufficiently low, and at the retail level when the degree of downstream competition is higher. However, when governments require a level or revenue sufficiently high (that is, above a certain threshold determined by the degree of downstream competition), they choose a mix of upstream and downstream taxation. If products are homogeneous, the optimal policy is to tax only at the downstream level, irrespective of the revenue requirement.

Comparing the results of this model with the Leviathan case (Cont and Fernández Felices, 2016) the main conclusion is that, for a relatively high revenue requirement, welfarist governments tend to behave qualitatively similar to Leviathan governments: they both choose a combination of taxes if products have some degree of heterogeneity, whereas they rely on downstream taxation when products are homogeneous.

Some final observations are in order. Firstly, we focus the analysis on provincial governments within a country, but the main problem applies to national governments within a union. In particular, successive taxation was not considered so far in the analysis of origin vs destination principles. Secondly, it is important to stress that imports and exports of tax bases can play an important role on tax policies by subnational governments, to the point that this effect should not be neglected and need to be balanced against the typical negative effects on welfare associated with these tax schemes. This effect is sufficiently important for provinces in Argentina to rely on the *impuesto sobre los ingresos brutos* as their main source of revenue, or for some states in the US to be reintroducing gross revenue taxes on their tax portfolio again.

6 Appendix

6.1 Appendix A

Let the coefficients in (2)-(6) be as follows:

$$\alpha = \frac{3(1+\gamma)(2+3\gamma)+2\gamma(1+\gamma)^2}{2\left[2(2+3\gamma)+\gamma(1+\gamma)\right](2+\gamma)} \quad , \quad \beta = \frac{(1+\gamma)(2+3\gamma)}{2\left[2(2+3\gamma)+\gamma(1+\gamma)\right](2+\gamma)} \tag{15}$$

$$\Delta = \frac{1}{\left[2(2+3\gamma)+3\gamma(1+\gamma)\right]\left[2(2+3\gamma)+\gamma(1+\gamma)\right]} \quad , \quad \theta = \gamma(1+\gamma)\lambda \tag{16}$$

$$\omega = 2[(2+3\gamma)+\gamma(1+\gamma)]^2\Delta \quad , \quad \delta = \frac{(2+3\gamma)}{2[2(2+3\gamma)+\gamma(1+\gamma)]} \tag{17}$$

$$\lambda = \left[(2+3\gamma)+\gamma(1+\gamma)\right]\Delta \quad , \quad \rho = \frac{(1+\gamma)(2\omega(1+\gamma)+\gamma\theta)}{(2+\gamma)(2+3\gamma)} \tag{18}$$

$$\mu = \frac{(1+\gamma)}{4(2+\gamma)}\frac{\omega}{\delta}+\frac{\gamma(1+\gamma)^2}{(2+\gamma)}\lambda \quad , \quad \kappa = \frac{\gamma(1+\gamma)^2}{2(2+\gamma)(2+3\gamma)}\omega \tag{19}$$

$$\sigma = \frac{(1+\gamma)(2\theta(1+\gamma)+\gamma\omega)}{(2+\gamma)(2+3\gamma)} \quad , \quad \psi = \frac{(1+\gamma)[(2+3\gamma)+\gamma(1+\gamma)]}{(2+\gamma)\left[2(2+3\gamma)+\gamma(1+\gamma)\right]} \tag{20}$$

From section 2.3 we find FOC for p_A and p_B and solve for equilibrium prices. Let $\Lambda = a - c_R - c_P$. The constant term (other than those multiplying tax rates) is

$$\Phi = \delta\left(2a - 2c_R - 2c_P\right) + c_P = 2\delta\Lambda + c_P \tag{21}$$

so that $\Phi - c_P = 2\delta\Lambda$.

Replacing p_A and p_B into p_{1A} to p_{2B} we get retailers prices. The constant term (other than those multiplying tax rates) is

$$\Psi = \frac{a}{2+\gamma} + \frac{1+\gamma}{2+\gamma}(\Phi + c_R) \tag{22}$$

so that $\Psi - \Phi - c_R = \frac{a-\Phi-c_R}{2+\gamma} = \frac{\psi}{1+\gamma}\Lambda$.

Finally, replacing prices into x_{1A} to x_{2B} we quantities. The constant term (other than those multiplying tax rates) is

$$\Gamma = \alpha a - \beta a - \psi(c_R + c_P) = \psi\Lambda \tag{23}$$

6.2 Appendix B

First-order conditions (8) and (9) are linear in t and τ. The intersection of both equations provide a linear relationship between both tax rates, as in (12), where

$$F_{\Lambda} = \frac{2\psi\left(\frac{4\alpha\psi}{1+\gamma} + 4\delta\psi + 2\left(1-\alpha-\delta\right)\psi\right) - 2\psi\left(\frac{3\psi^2}{1+\gamma} - 4\delta\mu - 2(1-\omega)\psi\right)}{3\psi\left(\frac{4\alpha\psi}{1+\gamma} + 4\delta\psi + 2\left(1-\alpha-\delta\right)\psi\right) - 2(\alpha+\psi)\left(\frac{3\psi^2}{1+\gamma} + 4\delta\mu + 2(1-\omega)\psi\right)} \tag{24}$$

$$F_{\tau} = \frac{2(\mu+\psi)\left(\frac{4\alpha\psi}{1+\gamma} + 4\delta\psi + 2\left(1-\alpha-\delta\right)\psi\right) - 3\psi\left(\frac{3\psi^2}{1+\gamma} + 4\delta\mu + 2(1-\omega)\psi\right)}{3\psi\left(\frac{4\alpha\psi}{1+\gamma} + 4\delta\psi + 2\left(1-\alpha-\delta\right)\psi\right) - 2(\alpha+\psi)\left(\frac{3\psi^2}{1+\gamma} + 4\delta\mu + 2(1-\omega)\psi\right)} \tag{25}$$

6.3 Appendix C

In this Appendix we provide examples of Proposition 1 for different values of the product differentiation parameter($\gamma = 0$, $\gamma = 1$, $\gamma = 10$ and $\gamma \to \infty$).

Case: $\gamma = 0$. Without downstream competition, equations (11) and (12) are:

$$t + \tau = \frac{\Lambda - \sqrt{\Lambda^2 - 8R}}{2} \qquad ; \qquad t = -\frac{1}{4}\Lambda + \frac{19}{16}\tau$$

The solution is

$$(t,\tau) = \begin{cases} \left(0, \frac{\Lambda - \sqrt{\Lambda^2 - 8R}}{2}\right) & if \quad R < \frac{30}{361}\Lambda^2 \\ \left(\frac{11}{70}\Lambda - \frac{19}{70}\sqrt{\Lambda^2 - 8R}, \frac{24}{70}\Lambda - \frac{16}{70}\sqrt{\Lambda^2 - 8R}\right) & if \quad \frac{30}{361}\Lambda^2 \leq R < \frac{1}{8}\Lambda^2 \\ \left(\frac{11}{70}\Lambda, \frac{24}{70}\Lambda\right) & if \quad R = \frac{1}{8}\Lambda^2 \end{cases}$$

Case: $\gamma = 1$. Equations (11) and (12) simplify to

$$t + \tau = \frac{\Lambda - \sqrt{\Lambda^2 - \frac{36r}{7}}}{2} \qquad ; \qquad t = \frac{14}{265}\Lambda + \frac{551}{903}\tau$$

Therefore the solution is

$$(t,\tau)=\begin{cases}\left(\frac{\Lambda-\sqrt{\Lambda^2-\frac{36}{7}R}}{2},0\right) & if \quad R<\frac{7}{36}\left(1-\left(\frac{214011}{239295}\right)^2\right)\Lambda^2 \\ & \sim 0.0389\Lambda^2 \\ \left(\frac{171299}{770620}\Lambda-\frac{551}{2908}\sqrt{\Lambda^2-\frac{36}{7}R},\frac{214011}{770620}\Lambda-\frac{903}{2908}\times\sqrt{\Lambda^2-\frac{36}{7}}\right) & if \quad 0.0389\Lambda^2\leq R<\frac{7}{36}\Lambda^2 \\ \left(\frac{171299}{770620}\Lambda,\frac{214011}{770620}\Lambda\right) & if \quad R=\frac{7}{36}\Lambda^2\end{cases}$$

Case: $\gamma=10$. Equations (11) and (12) become:

$$t+\tau=\frac{\Lambda-\sqrt{\Lambda^2-2.67R}}{2} \qquad ; \qquad t=0.38\Lambda-0.04\tau$$

Therefore the solution is

$$(t,\tau)=\begin{cases}\left(\frac{\Lambda-\sqrt{\Lambda^2-2.67R}}{2},0\right) & if \quad R<0.35\Lambda^2 \\ \left(0.38\Lambda+0.02\sqrt{\Lambda^2-2.67R},0.12\Lambda-0.52\sqrt{\Lambda^2-2.67R}\right) & if \quad 0.35\Lambda^2\leq R<0.37\Lambda^2 \\ (0.38\Lambda,0.12\Lambda) & if \quad R=0.37\Lambda^2\end{cases}$$

Case: $\gamma\to\infty$. This case corresponds to a situation with homogeneous products at the downstream level. Equations (11) and (12) turn out to be:

$$t+\tau=\frac{\Lambda-\sqrt{\Lambda^2-2R}}{2} \qquad ; t=\frac{1}{2}\Lambda-\frac{1}{4}\tau$$

Therefore the solution is $(t,\tau)=(\frac{\Lambda-\sqrt{\Lambda^2-2R}}{2},0)$ for all $R\leq 1/2.\Lambda^2$.

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